(d) The fuel consumption, $f$, of a car, in litres per 100 km, is related to the velocity, $v$, in km h$^{-1}$, by the formula

$$f = 16 - 0.2v + \frac{v^2}{250}$$

Show that the fuel consumption is a **minimum** when $v = 25$ km h$^{-1}$.

91262 2012

(e) A train starts from rest.
Its acceleration $t$ seconds after it starts is given by

$$a = \frac{1}{4}(20 - t)\text{ m s}^{-2}$$

What distance does the train cover in the first 30 seconds?

(c) $g(x) = x^3 - 9x^2 + 24x - 8$

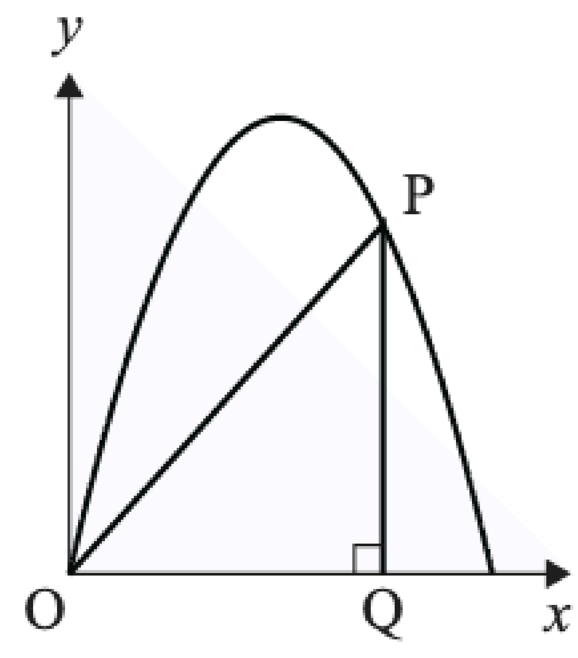
For what values of $x$ is $g$ a decreasing function?

You must justify your answer, using calculus.

(d) A right angled triangle OPQ is drawn as shown where O is at (0,0).

P is a point on the parabola $y = ax - x^2$

and Q is on the $x$-axis.

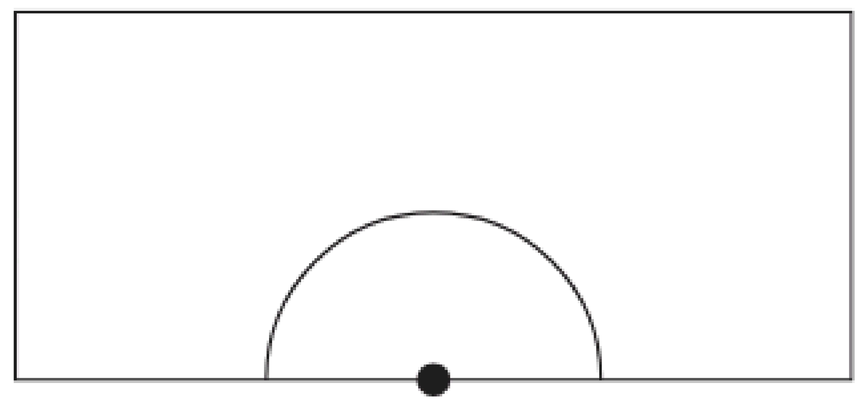



Show that the maximum possible area for the triangle OPQ is $\frac{2a^3}{27}$

(d) The curve of $f(x) = 2x^3 + \mathrm{A}x + \mathrm{B}$ has a tangent with gradient 10 at the point $(-2,33)$.

Find the coordinates of the point on the curve where $x = 4$.

(e) A chemical is dropped into the water in a rectangular swimming pool at a point half-way along its length.



After 0.1 minutes, the chemical spreads in a semi-circular shape with radius $r$ metres, where

$$r = 1 + 2t$$

and $t$ is the time in minutes since the chemical was added to the water.

When the chemical first reaches the far side of the pool, the area of the semi-circle is increasing at the rate of $60\ \text{m}^2\ \text{min}^{-1}$.

Find the width of the pool. (Area of a circle $= \pi r^2$)

91262 2012

(e) $g(x) = -x^3 + 3x + 2$

For what values of $x$ is $g$ a decreasing function?

You must **show the use of calculus** in your working.

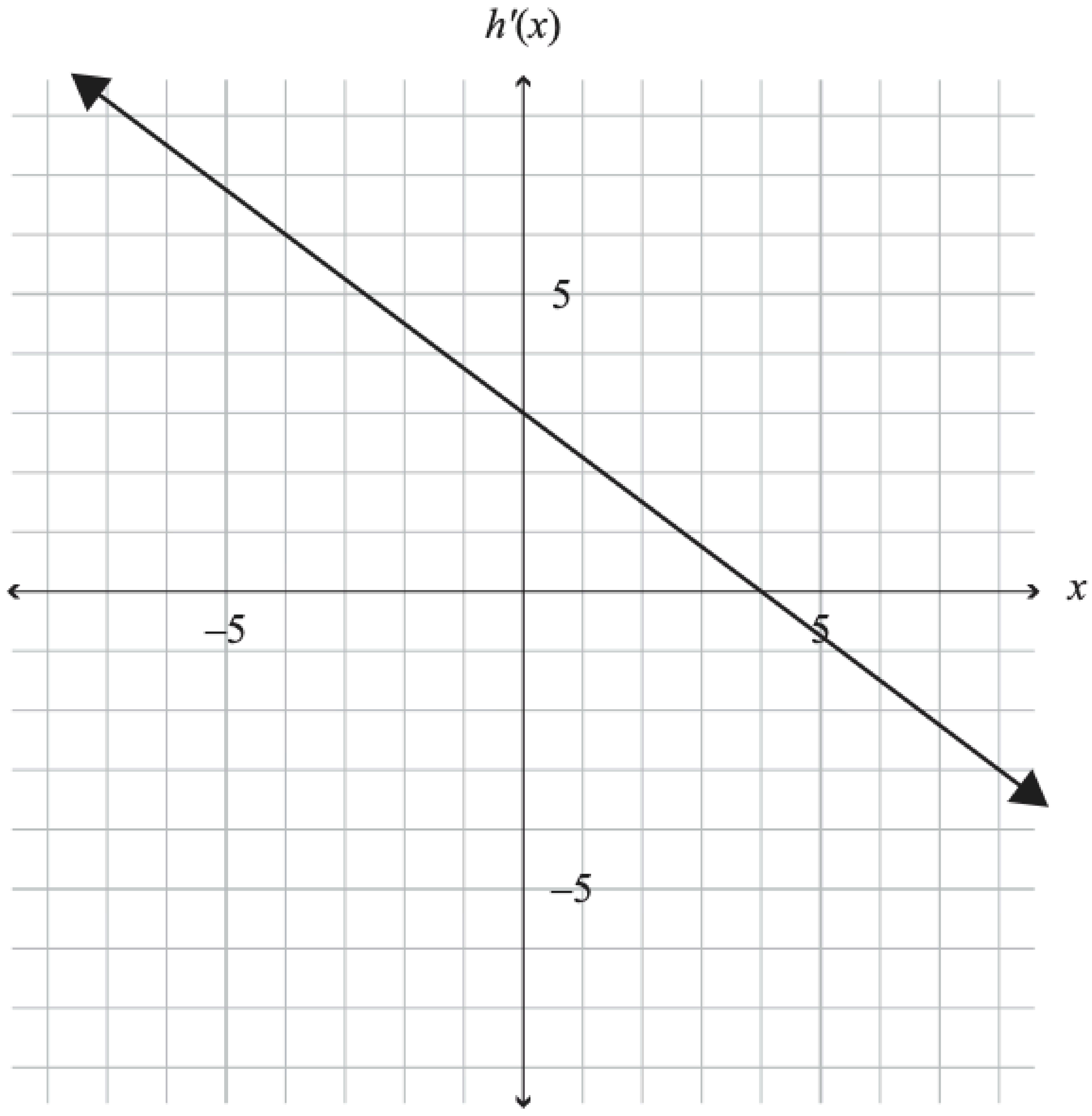
(f) A curve has gradient function $f'(x) = mx + 2$. The curve passes through the points (2,10) and (–1,–8).

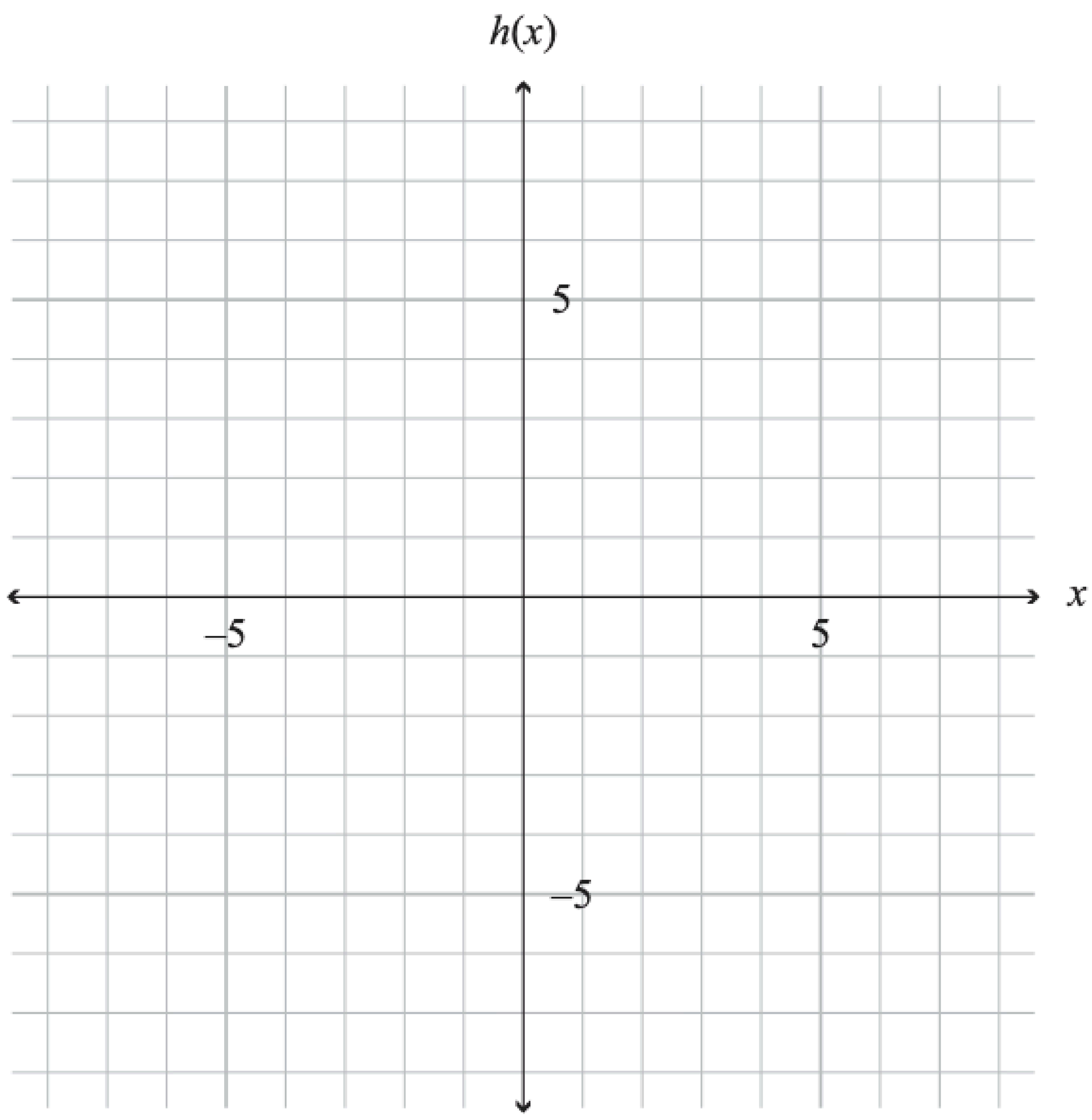
Find $f(x)$, the equation of the curve.

91262 2013

(d) Sketch the function $h(x)$ for the gradient function $h'(x)$ below, given that the maximum value of $h$ is 5.

Show the vertex clearly.



(e) The gradient of a curve is given by $\frac{dy}{dx} = 6x^2 - 12x$.

The $y$-coordinate of the minimum turning point of the curve is 10.

Find the equation of the curve.

(f) A car is travelling at a constant speed until the car's brakes are applied.

The car's speed changes at a rate given by $-0.08t$ metres $\sec^{-2}$ after the brakes are applied, where $t$ sec is the time since the brakes were applied.

3 seconds after the brakes are applied, the speed of the car is 5 metres $\sec^{-1}$.

How far will the car travel with the brakes applied before it stops?

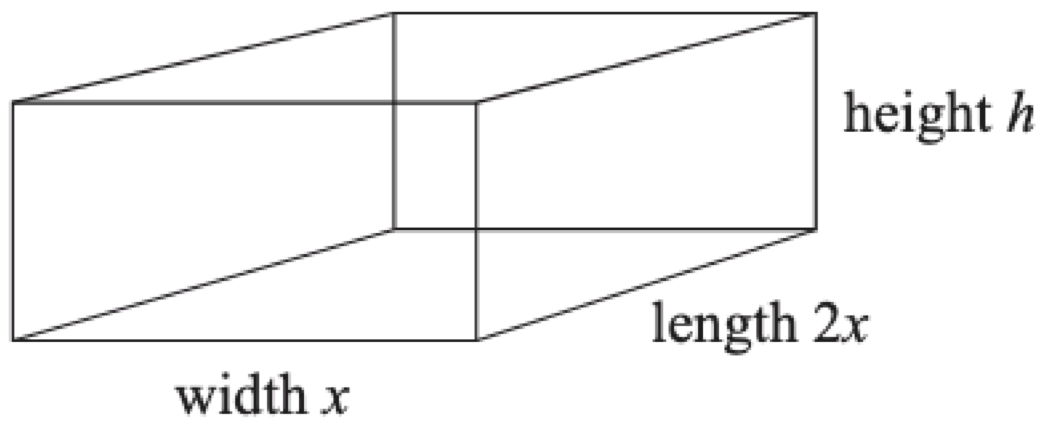
(d) The curve of $f(x) = \mathrm{P}x^2 + \mathrm{Q}x + 2$ has a turning point when $x = \frac{2}{3}$.
The curve passes through the point (1,9).

Find the coordinates of the point on the curve where $x = 3$.

91262 2013

(e) The frame of a crate is made up of 12 steel rods that have a total length of $L$ cm.

The length of the crate is twice the width.




Show that the length of the crate will be $\frac{L}{9}$ cm when the volume is a maximum.

91262 2013

(e) $g(x) = \frac{2x^3}{3} + \frac{3x^2}{2} - 20x + 4$

For what values of $x$ is $g$ a decreasing function?

*You must use calculus in finding your solution.*

91262 2014

(f) During a fund-raising cycle ride, the distance $s$ kilometres of a cyclist from a fixed point on his ride is modelled by the function

$$s(t) = 0.1875t^3 - 2.25t^2 + 21t + 0.5$$

where $t$ is the time in hours since the cyclist passed the fixed point.

How far will the cyclist be from the fixed point when he reaches his minimum speed?

(e) The gradient of the graph of a function is given by $\frac{dy}{dx} = 9x - 3x^2$.

At the maximum turning point of the graph of the function, $y = -4$.

Find the equation of the graph.

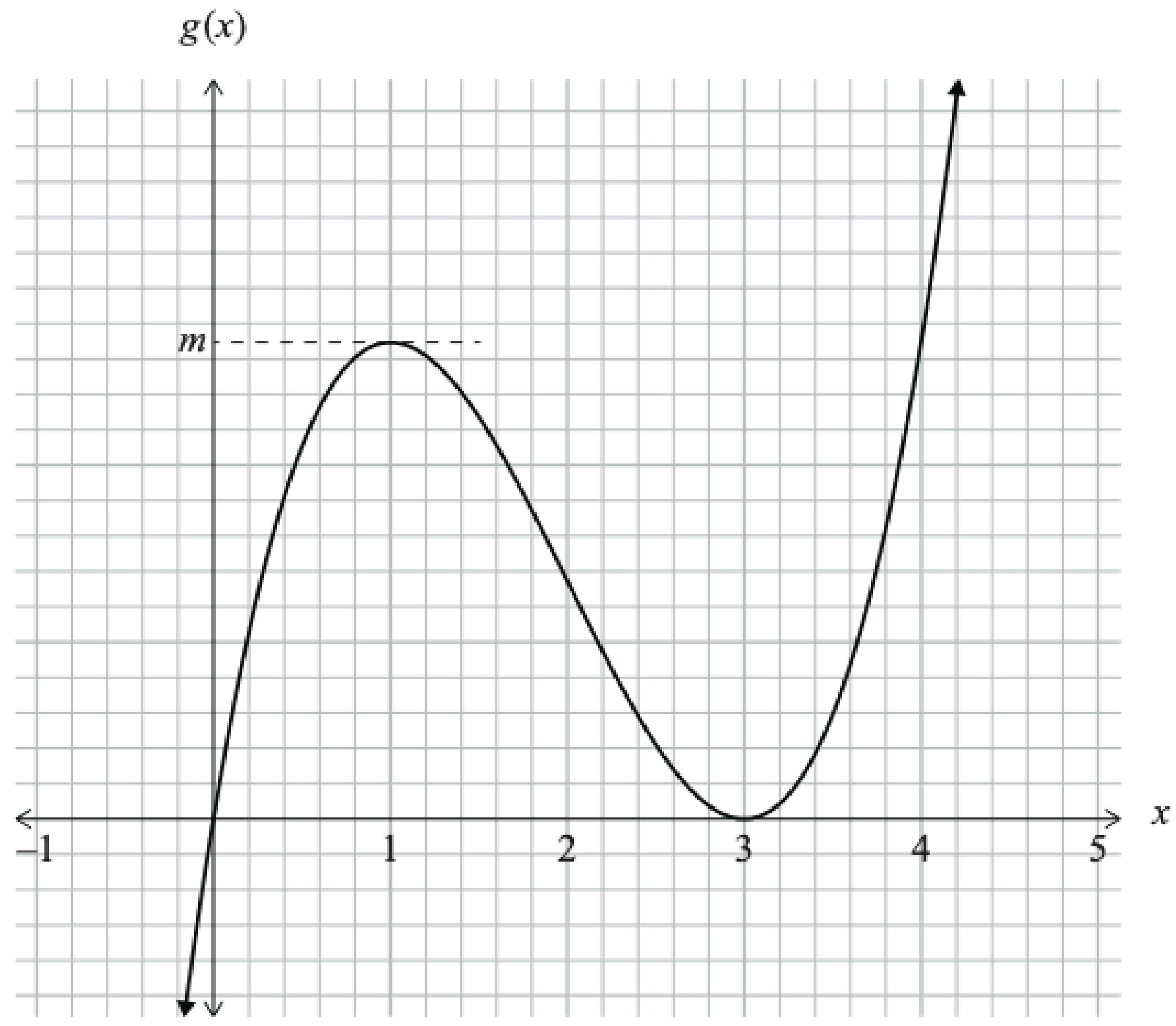
**Note:** The turning point where $y = -4$ must be shown to be a maximum.

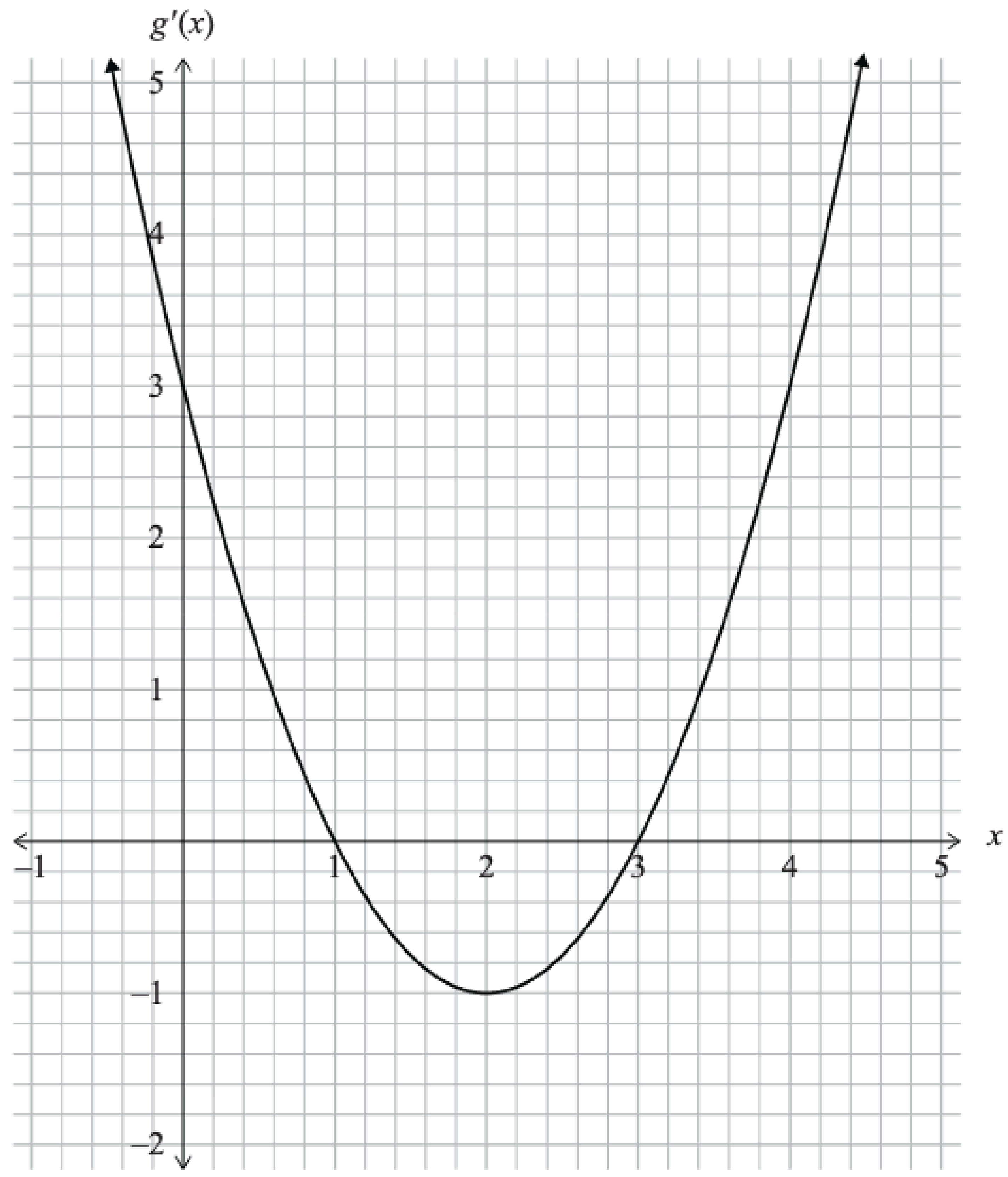
(f) The tangent to the graph of the function $y = px^2 + 4x - 5$ at the point where $x = 3$ passes through the point (0,4).

Find the value of $p$, and hence the equation of the tangent.

91262 2014

(e) The graph of a function $g(x)$ and the graph of its gradient function, are shown below.




Find the value $m$, the $y$-value for the maximum turning point of the function $g(x)$.

(f) An aircraft is travelling at 70 m $s^{-1}$ when it lands.

Its speed changes at a constant rate of $-3.3$ m $s^{-2}$.

**Use calculus** to find how far the aircraft will travel from where it lands to where it has a speed of 4 m $s^{-1}$.

91262 2014

(b) $f(x) = 8 + 3x + x^2 - \dfrac{x^3}{3}$

For what values of $x$ is $f$ a decreasing function?

Justify your answer.

*You must show the use of calculus.*

(d) A train passes a signal at a velocity of 40 m $s^{-1}$.
The train's acceleration, $a$ m $s^{-2}$, $t$ seconds after it passes the signal, can be modelled by the function

$$a(t) = (16 - 2t)$$

(i) What is the greatest speed attained by the train after it passes the signal?

(ii) How far past the signal does the train travel before it stops?

(c) A skateboard park has a mound that is $h$ metres high at the point where the horizontal distance, from a fixed point P, is $x$ metres.

The mound can be modelled by

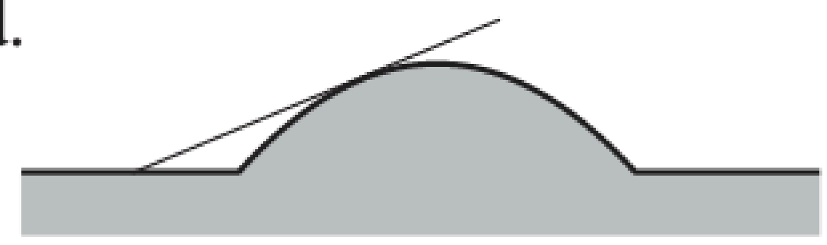
$$h = -0.5x^2 + 3x - 1.5$$


(i) What is the maximum height of the mound?

(ii) A ramp up the side of the mound is a tangent to the mound.

The ramp can be modelled by the function

$$h = 0.5x - c$$



Use calculus to find the vertical distance below the top of the mound where the ramp will meet the mound.

*Ignore the thickness of the ramp.*

Continued

(iii) The height $h$ metres of a skateboard path at a horizontal distance $r$ metres from another point Q, can be modelled by the function

$$h = \frac{r^3}{3} - 2r^2 + 3r \qquad (0.15 < r < 3.5)$$

There is a height regulation that requires no part of the skateboard path to be more than 3 m above the ground.

Fully describe this curve including its turning points, and state whether or not the skateboard path complies with the height regulation.

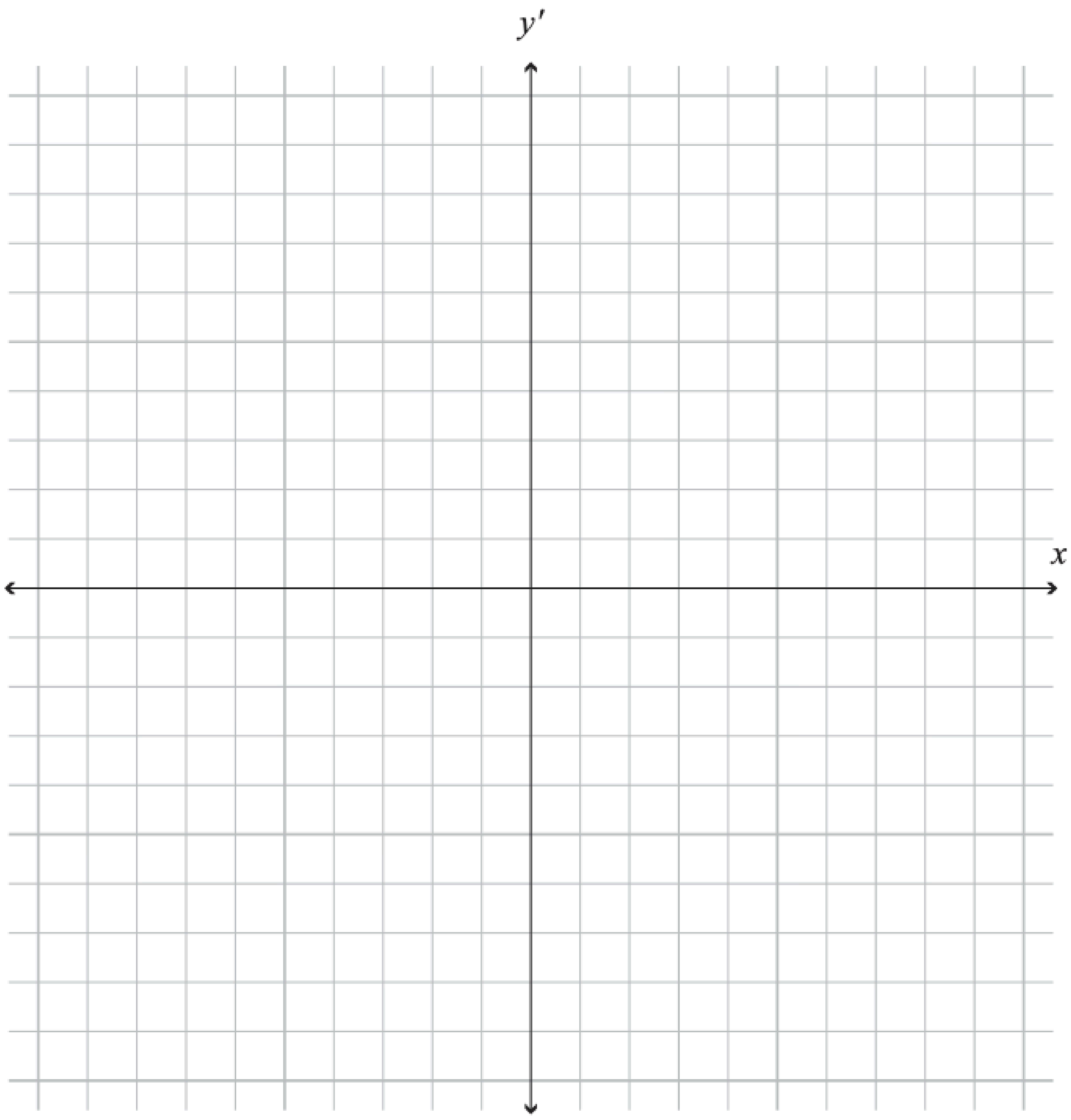
*You must show calculus in answering this question.*

(d) For a function $y = -ax^2 + bx + c$,

$a$, $b$, and $c$ are positive numbers and $b = 2a$.

On the grid below, sketch the gradient function.

Show the value of all intercepts. The $y'$-intercept should be given in terms of $b$.



(e) $y$ is the value of $x$ after 3 has been subtracted and then the answer doubled, and $x$ is between $-0.5$ and 3.

Find the maximum and minimum values of the product of $x^2y$.

*Justify your answer.*

(d) The equation of a function $y = f(x)$ has gradient function of the form $f'(x) = 2x - \text{a}$, where a is a constant.

The point (3,4) is the turning point on the graph of the function.

Find the equation of the function.

(e) Find the local minimum value of the function $y = x^3(x - 4)$.

Justify your answer.

91262 2016

(d) A chemical is slowly leaking onto a floor.

The chemical spreads out from the point where it lands in a shape that can be modelled by a circle of radius $r$ cm.

At a time $t$ seconds after the chemical leak is noticed, $r$ is given by $0.1t + 2$.

Use calculus to find the rate of change of area of the circle, with respect to time, when its radius is 10 cm.

(Area of circle = $\pi r^2$)

(e) A function is defined by $y = 3x^3 - 4a^2x + 5$ where $a$ is a positive number.

Find the range of values of $x$ in terms of $a$ for which the function is decreasing.

(c) Meg is riding her motocross bike.

When she passes a fixed point P on the track, she has a speed, $v$, of 5 m $s^{-1}$, and her acceleration, $a$, is 0.6 m $s^{-2}$.

(i) If she were to continue to accelerate at this rate, what is her speed when she has been riding for 10 seconds after passing P?

(ii) How far will she have travelled from P when she reaches a speed of 8 m $s^{-1}$ ?

Continued

(iii) Meg's friend Leo was riding with her, but he begins to decelerate when they reach a speed of 8 m s$^{-1}$.

If he decelerates at 0.2 m s$^{-2}$, how far past the point P will he be when he reaches a speed of 6 m s$^{-1}$?

(d) A tangent to the graph of the function $f(x) = 3x^2 - 4x$ has a gradient of 2, and passes through the point $(5,a)$, where $a$ is a constant.

Find the value of $a$.

(e) The function $f(x) = x^3 + ax^2 + bx + 2$ has turning points when $x = -1$ and $x = 3$.

Find the values of $a$ and $b$.

(d) Use calculus to find the value of $k$ if the line $y = 6x + k$ is a tangent to the graph of the function $f(x) = x^2 + 2x - 1$.

(e) Use calculus to prove that the graph of the function

$$y = x^3(3 - x)$$

has a local maximum when $x = \frac{9}{4}$.

Justify that the turning point is a local maximum.

(c) An object can move in either direction on a straight track and has a constant acceleration of $-4\text{ cm s}^{-2}$.

A fixed point P is marked on the track.

When a recording of the object's motion begins, the object:

- is 12 cm from P
- is moving away from P, and
- has a velocity of $6\text{ cm s}^{-1}$.

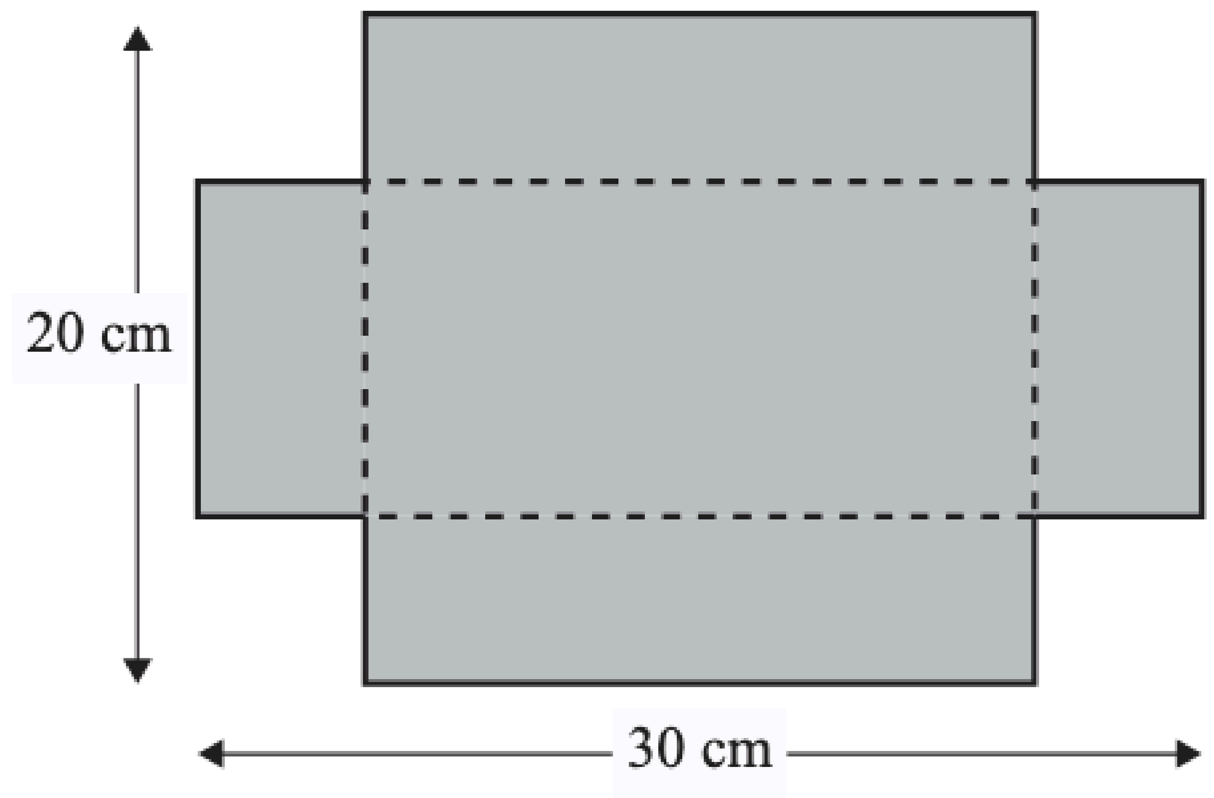
(i) Using calculus, find the speed of the object 5 seconds after its motion began being recorded.

(ii) What is the maximum distance of the object from the point P?

Justify that this is the maximum distance.

(d) Find the maximum volume of an open box (i.e. a box with a base and sides, but no lid) that can be made from a rectangular piece of cardboard measuring 20 cm by 30 cm, by removing the corner squares and folding along the dotted lines.

Justify that this is the maximum volume.

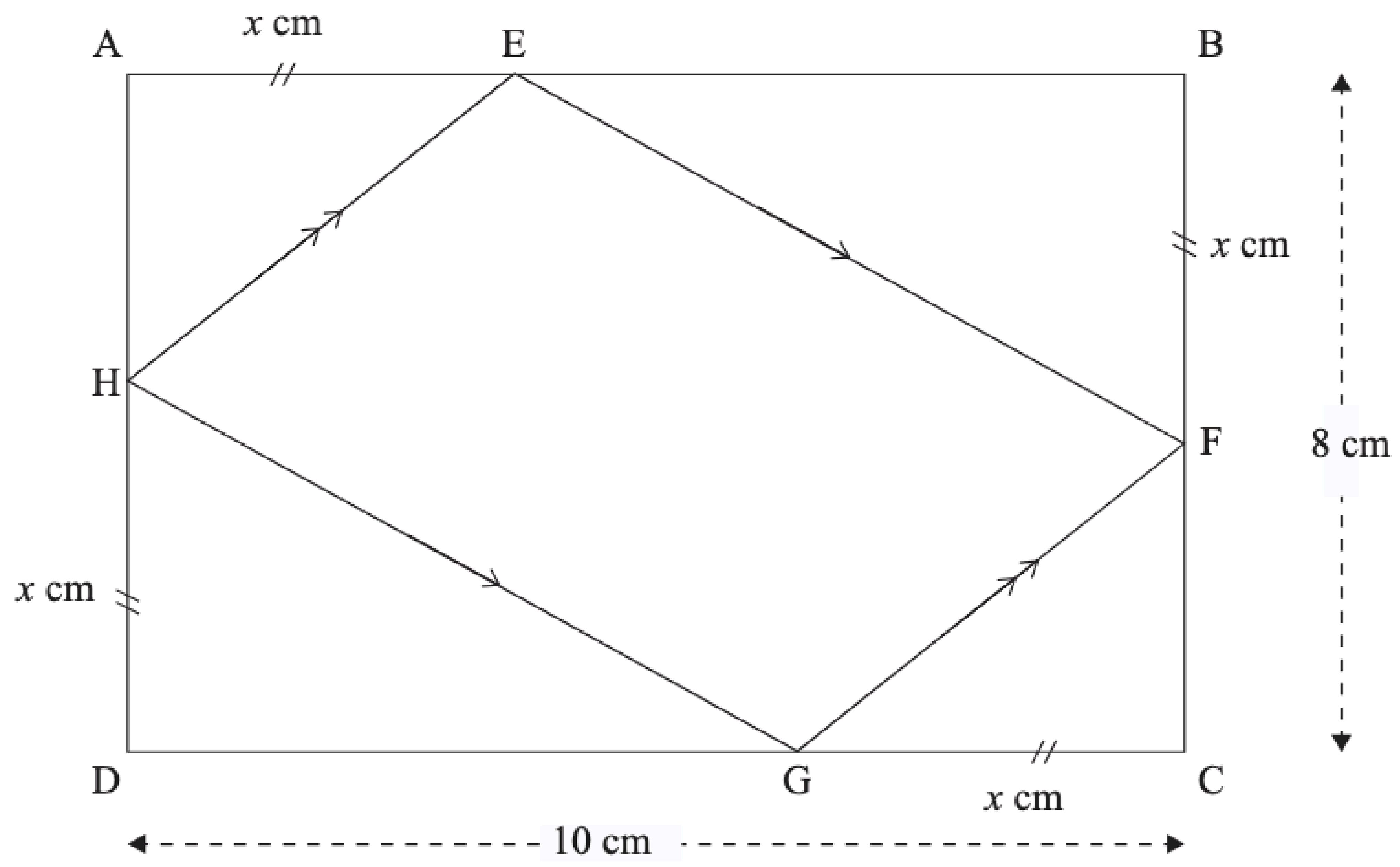


(d) Use calculus to find the values of $x$ for which the graph of the function $f(x)=\frac{2}{3}x^3+\frac{9}{2}x^2-5x-18$ is increasing.

(e) A rectangle ABCD measures 10 cm by 8 cm. A parallelogram EFGH can be drawn inside the rectangle, as shown in the diagram below.

Suppose that the distance from each corner of the rectangle to the next vertex of the parallelogram, in a clockwise direction, is $x$ cm.

That is, AE = BF = CG = DH = $x$.




Use calculus to find the smallest possible area that the parallelogram can have.

Justify that your answer is a minimum.

(c) Adam is operating his drone. It is moving in a straight line and $t$ seconds after passing a tree its acceleration, $a$ m s$^{-2}$ , is given by

$$a(t) = 6 - 12t.$$

Two seconds after the drone passed the tree, its velocity was 20 m s$^{-1}$.

How far was the drone from the tree when its velocity was 20 m s$^{-1}$ ?

(e) The graph of the function $y = x^3 - 6x^2 + kx - 5$ has a turning point at $x = 3$.

Use calculus methods to find the coordinates of both turning points.

Determine the nature of each turning point, justifying your answer.

(d) A tangent to the graph of the function $y = -\frac{1}{3}x^3 + kx + 4$

at a certain point P, has gradient of –7 and intersects the graph again at (–6,64).

Use calculus to find the co-ordinates of the point P.

(b) Find the coordinates of the point(s) on the graph of the function $y = 4x^3 - 4x + 4$ where the tangent to the curve is parallel to the line $y - 8x + 6 = 0$.

(d) Use calculus to find the quadratic function of $x$ that has the following properties:

- a rate of increase (gradient) of 7 when $x = 0$
- a turning point when $x = 1$
- a value of $-20$ when $x = 4$.

(e) The graphs of

$$g(x) = x^3 - ax^2 + 6 \quad \text{and} \quad h(x) = 2x^2 + bx + 13$$

just touch when $x = -1$ (so they have a common tangent at the point of contact).

Use calculus to find the coordinates of the point of contact of the two graphs.

(c) Consider the graph of the function $f(x) = -2k^2x^3 + 3kx^2 + 12x - 55$, where $k$ is a positive constant.

Use calculus to find expressions, in terms of $k$, for the range(s) of values of $x$ for which this graph is increasing.

Justify your choice of range(s) clearly.

(e) A car travelling at a constant speed of 28 m $s^{-1}$ on a straight road is approaching a corner. The driver applies the brakes and decelerates at a constant rate of 4 m $s^{-2}$ until the car reaches the corner with a speed of 10 m $s^{-1}$.

Use calculus to find how far the car was from the corner when the driver first applied the brakes.

Justify your answer.

(b) A parcel in the shape of a rectangular cuboid with a square cross section is to be sent through the post. The sum of the length of the cuboid and the perimeter of the square cross section is to be 100 cm.

*Diagram is NOT to scale*

Find the maximum possible volume of the parcel.

Explain how you know that your answer is the maximum, not the minimum, volume.

(c) There are two points, A and B, on the graph of the function $f(x) = x^3 - 3x^2 - 4x$ where the tangent to the graph passes through the origin.

Find the coordinates of points A and B and the equation of each tangent.

(e) A fishing boat is 80 km from its port when it reaches its fishing grounds.

Having reached its fishing grounds, the boat accelerates in a straight line directly away from its port as it catches fish. The acceleration of the boat is given by $a(t) = 0.5 \text{ km h}^{-2}$, where $t$ is the number of hours since it started fishing.

The speed of the boat is $3 \text{ km h}^{-1}$ when it starts fishing.

During which hour did the boat travel 11.75 km? You must use calculus to find your answer.

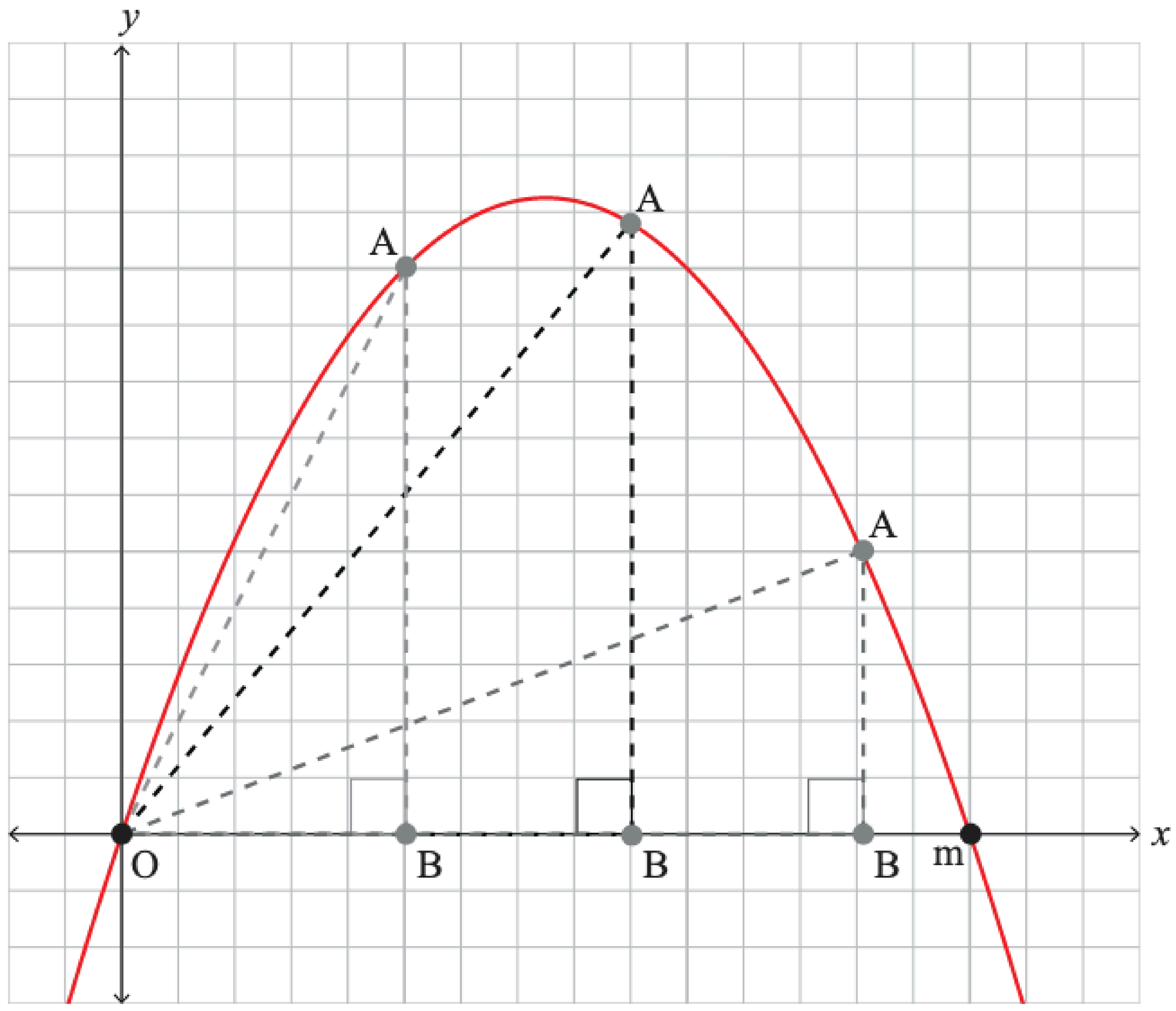
(e) A cubic function, $f(x) = ax^3 + bx^2 + cx + d$, has turning points that occur at $x = 3$ and $x = -5$.

Find an expression for $c$ in terms of $a$.

(e) A right-angled triangle OAB is drawn within a parabola, as shown in the diagram below. Three possible triangles OAB have been drawn.

The point O is the origin (0,0), and point A can lie anywhere on this parabola above the $x$-axis.

The equation of the parabola is of the form $y = mx - x^2$, where $m$ is a positive constant.




Use calculus to find an expression, in terms of $m$, for the maximum possible area of triangle OAB.

(c) The number of daily viewers for a new presenter on a video gaming streaming service can be modelled by the following equation:

$$V = -11t^2 + 528t \ \{0 \leq t \leq 48\}$$

Where $V$ represents the number of daily viewers and $t$ represents time in months.

(i) Find the rate(s) of change of daily viewers, with respect to $t$, when the number of daily viewers is 3520.

(ii) What is the maximum number of daily viewers that this new presenter gets during the first 48 months?

Continued

(iii) The number of daily viewers for an experienced presenter on a video gaming streaming service can be modelled by the following equation:

$$V = 1.6t^3 - 130t^2 + 2900t \ \{0 \leq t \leq 48\}$$

Where $V$ represents the number of daily viewers and $t$ represents time in months.

Once the stream reaches 10 000 daily viewers, it becomes monetised (the presenter earns money). If the daily viewership falls below 10 000, the presenter will lose this income and will no longer make any money.

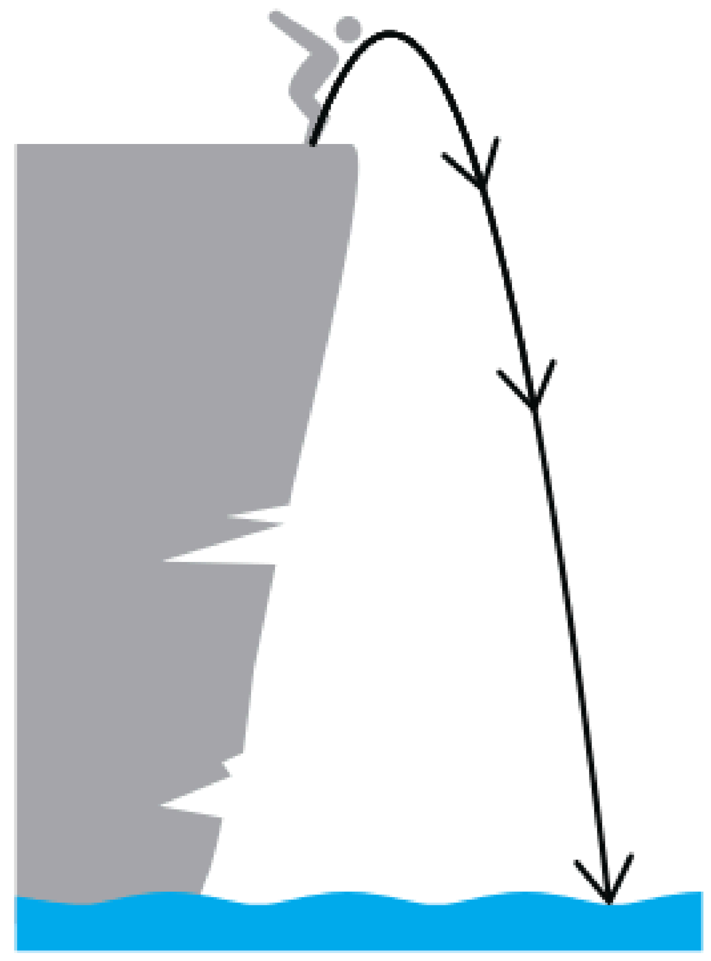
Use calculus methods to determine if the stream, after it becomes monetised, ever stops earning money for the presenter.

(c) (i) A cliff diver jumps up into the air above a cliff and then falls down into the water below.

Their acceleration is constant at $-9.8 \text{ m s}^{-2}$.

The diver jumps up with an initial vertical velocity of $2.8 \text{ m s}^{-1}$.

Using calculus methods, find the velocity of the diver one second after they jumped.



Continued

(ii) The diver hits the water at a velocity of $-22.68 \text{ m s}^{-1}$ (negative velocities indicate the diver is moving down).

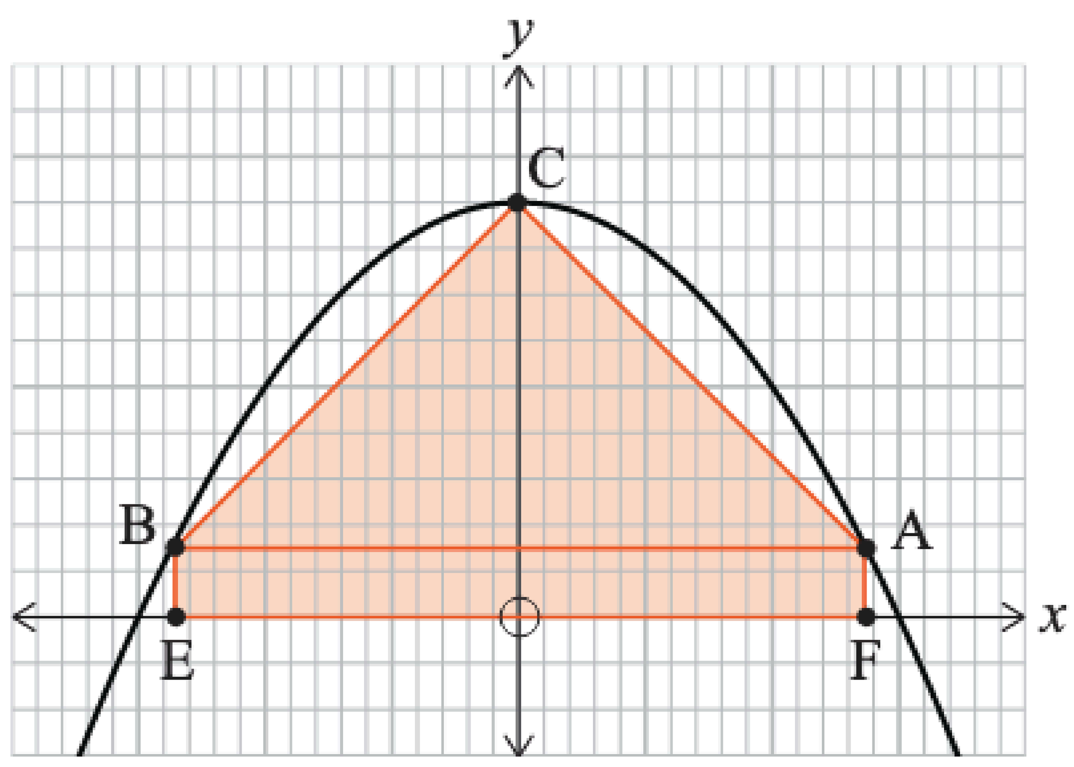
Find the maximum height (above the water) that the diver reached.

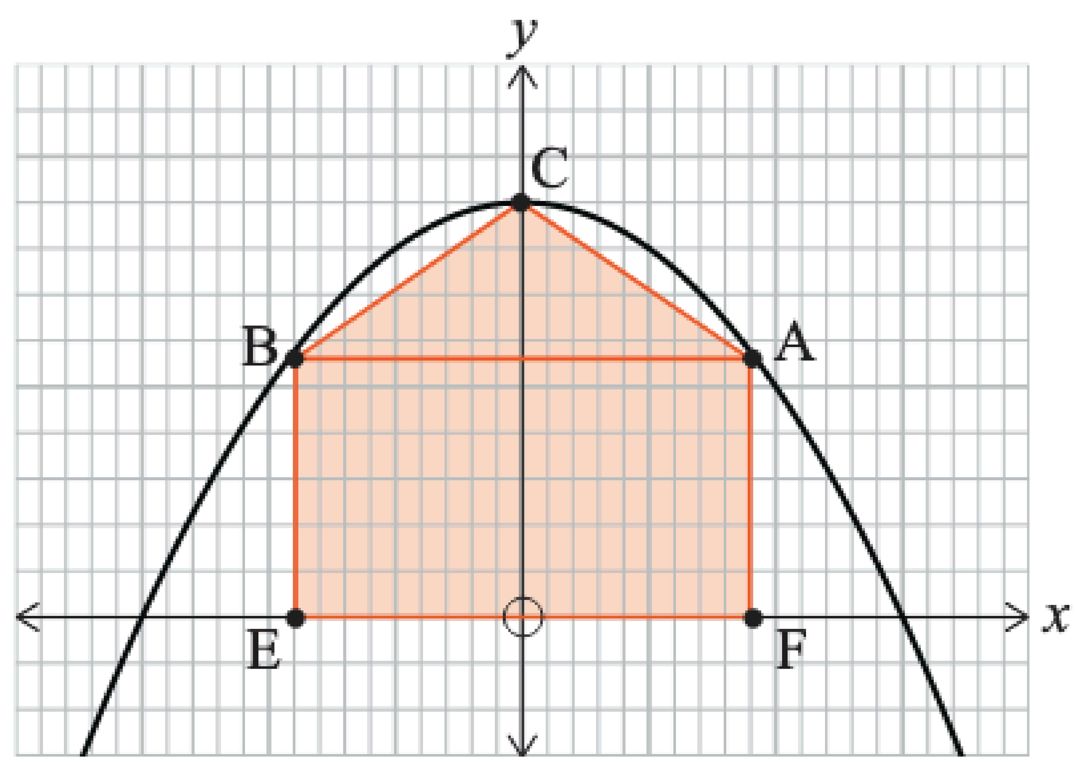
You may wish to begin by finding the height of the cliff above the water.

(d) Two possible configurations of a house shape drawn below a parabola are shown below.

A house shape is drawn as shown where:

- C is at (0,9).
- A and B are points on the parabola $y = 9 - x^2$.
- E and F are on the $x$-axis.



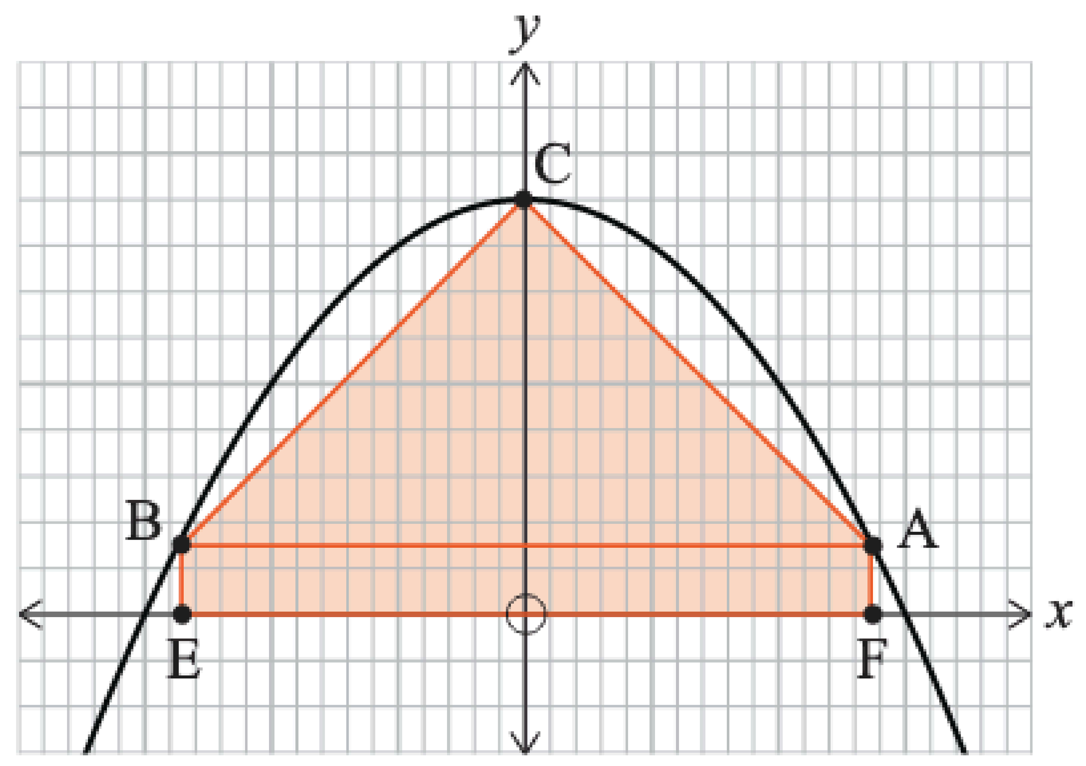




(i) Find the height of the wall (AF or BE) when the area of the house shape is a maximum. Justify that this is the maximum area.

Continued

(ii) For the generalised problem where the house shape is bounded by a parabola where:

- C is at $(0,d)$.
- A and B are on the parabola $y = d - kx^2$.
- E and F are on the $x$-axis.




Show that the maximum area enclosed by the house shape occurs when the area of the rectangular section ABEF and the area of the triangular section ABC are equal.

(d) The tangent to the curve $f(x) = \mathrm{p}x - \mathrm{q}x^2$ at the point $(2, -10)$ has a gradient of $-6$.

Find the values of the constants p and q.

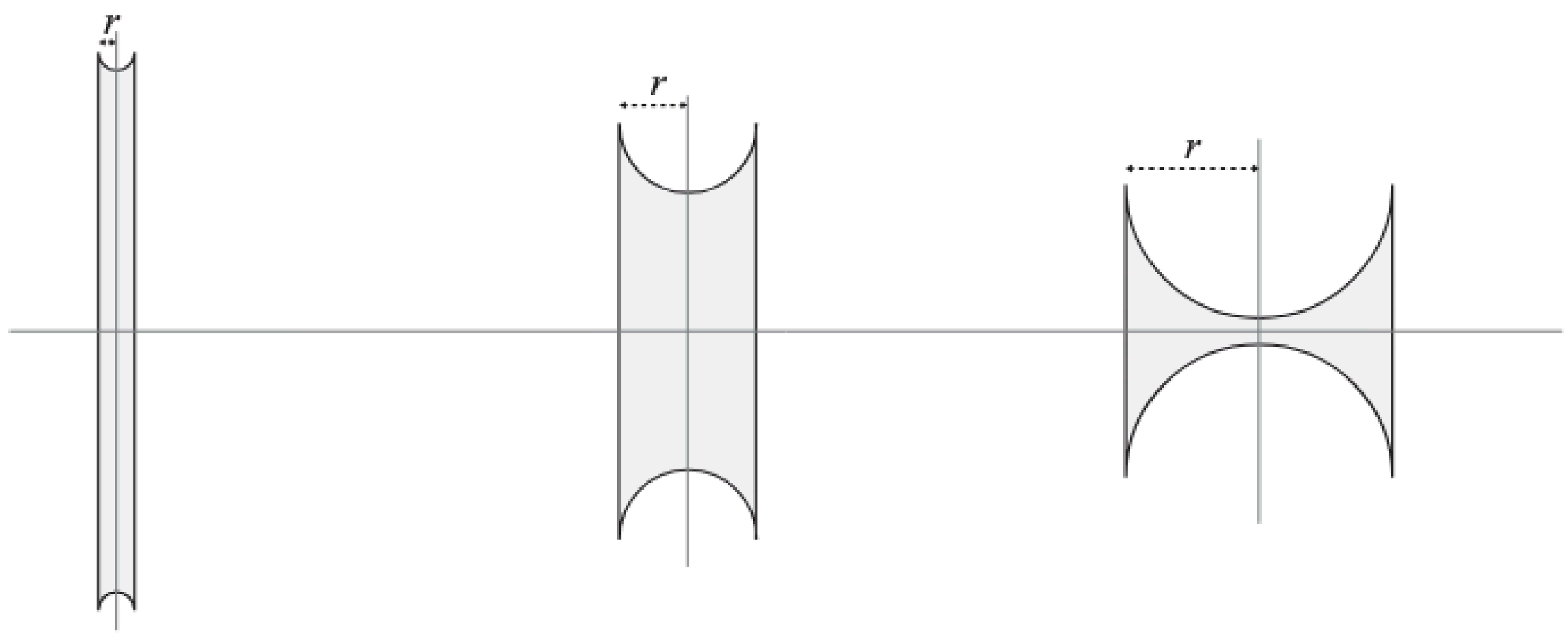
91262 2022

(e) A new business is designing a logo based on a stylised 'H' shape, as seen in the diagrams.

The logo is to be designed with two semi-circles and two straight lines. The owner of the new business wants to include space for the company's name inside the shape, so he has asked the designer to maximise the area inside the shape.

The owner plans to build a replica of the logo, and due to material constraints, the total perimeter of the shape must not be greater than 80 cm.

Three examples are shown here:



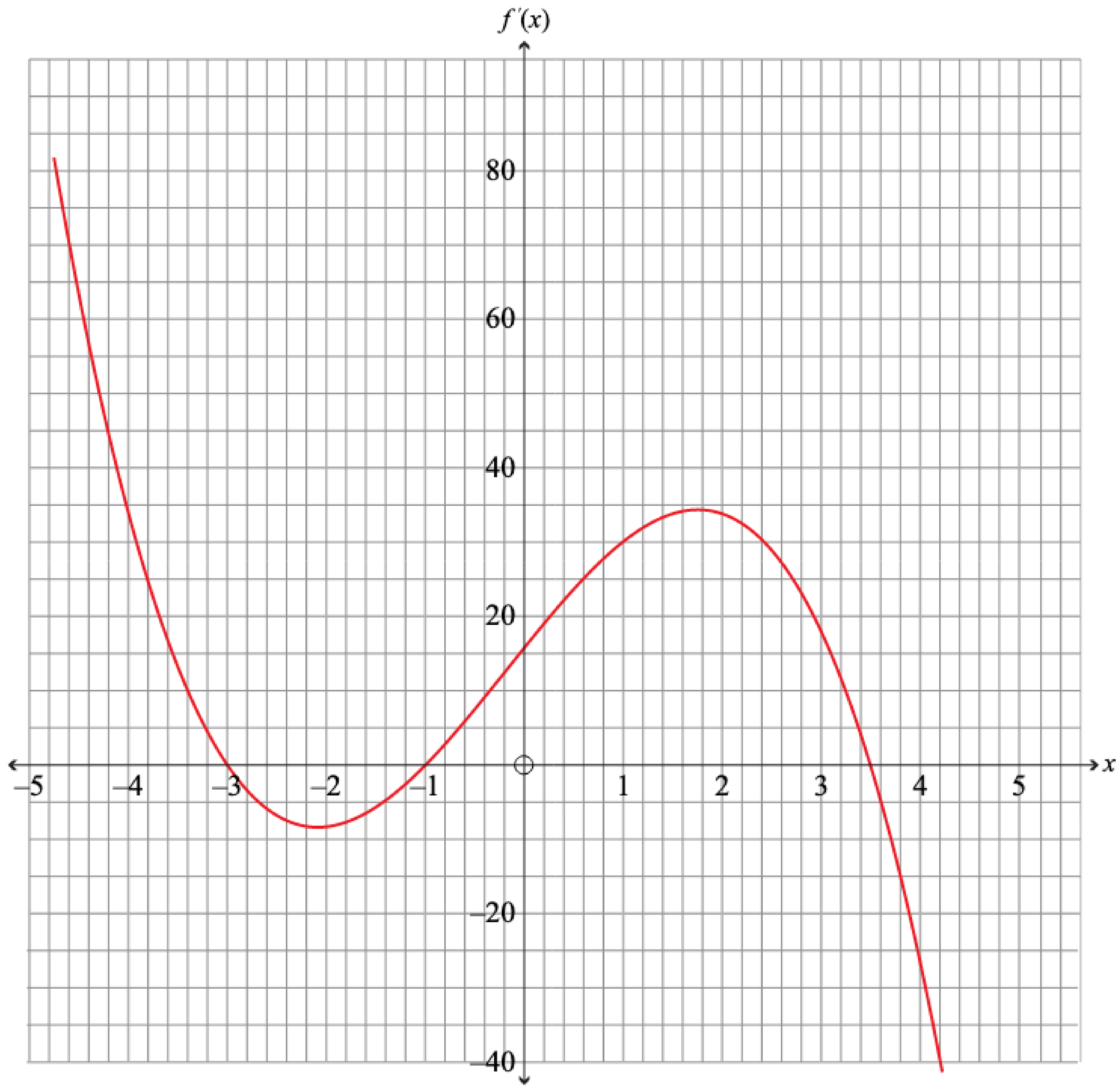

(i) Find the maximum area.

(ii) Use calculus methods to show that this is a maximum.

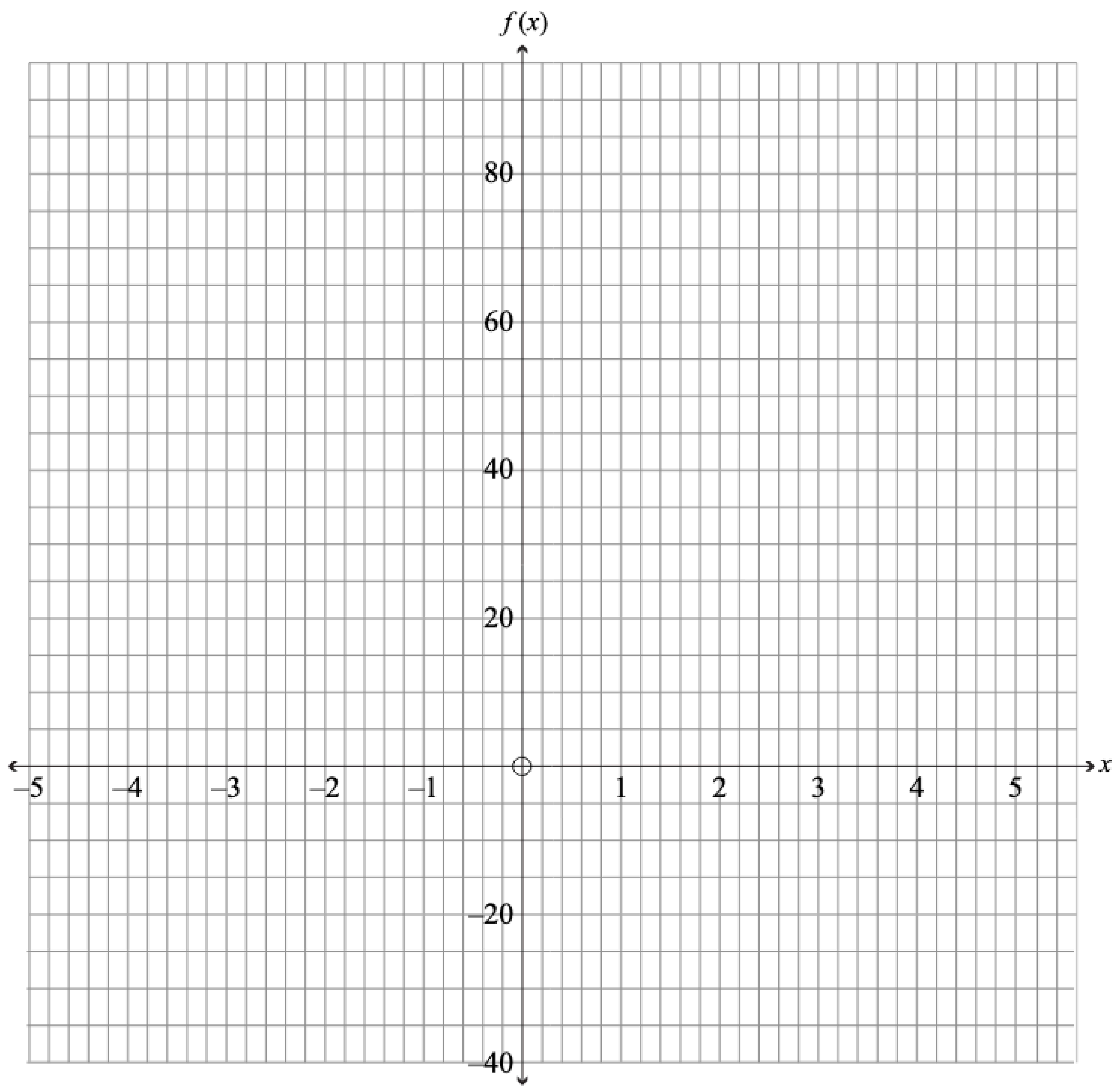
Circumference of a circle: $C = 2\pi r$

Area of a circle : $A = \pi r^2$

(a) The gradient graph of a function $y = f(x)$ is shown on the axes below.




Sketch a possible graph of the original function $y = f(x)$ on the axes below.



(c) (i) During the cricket match, spectators both come and leave the venue. The number of spectators can be modelled by the following equation:

$$P = 100t^2 - 2t^4 + 750 \quad (0 \leq t \leq 7.5)$$

where $P$ represents the number of spectators in attendance, and $t$ represents the time in hours since the start of the match.

Show that the rate of change of spectators is –528 people per hour when $t = 6$.

(ii) What is the meaning of this value for the rate of change?

Continued

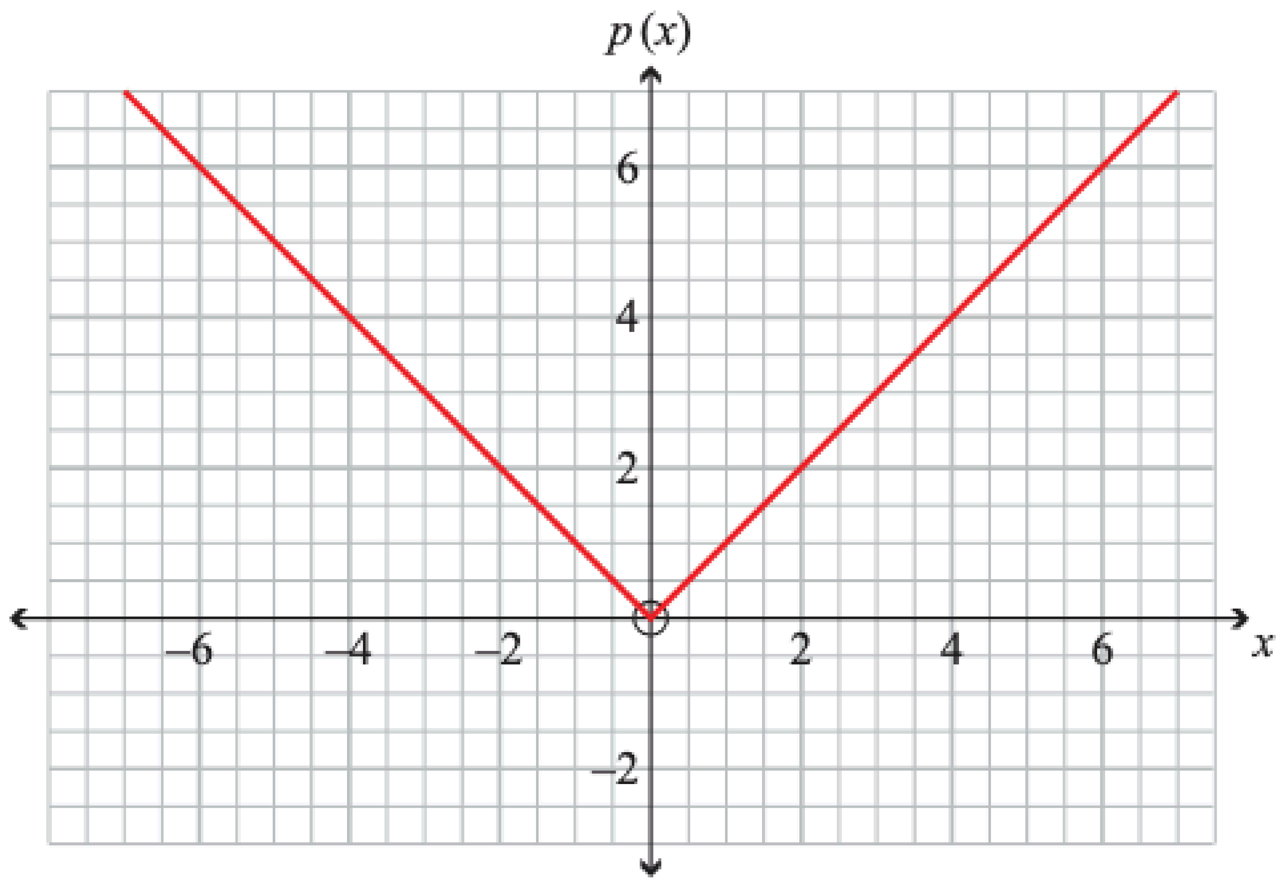
(iii) At another match, the number of spectators is modelled by the equation $P = kt^2 - 2t^4 + 750$, where $k$ is a constant.

The number of spectators is growing fastest when $t = 4$.

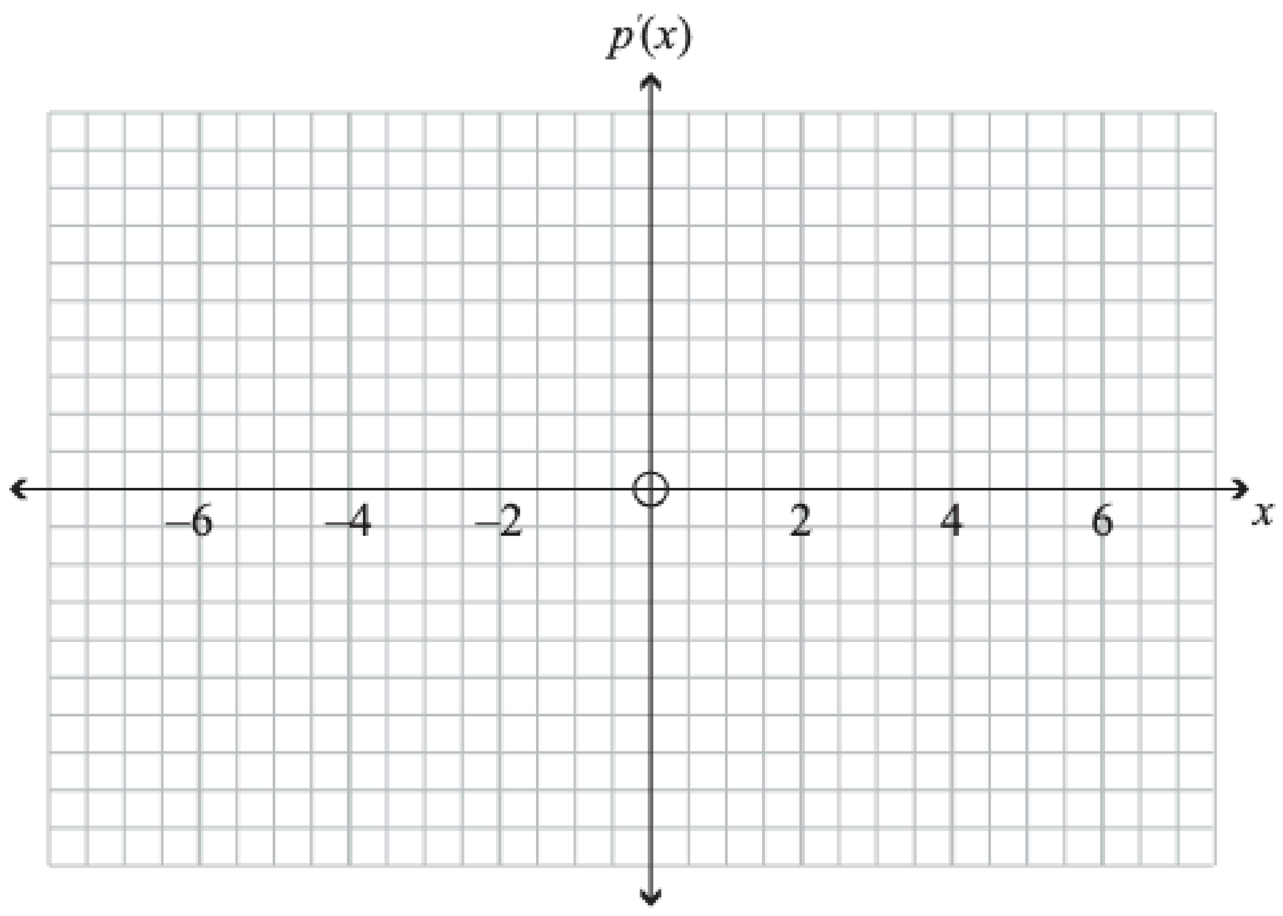
Find the value of $k$.

Explain your reasoning clearly, using correct mathematical statements.

(ii) The diagram below shows the graph of the function $y = p(x)$.



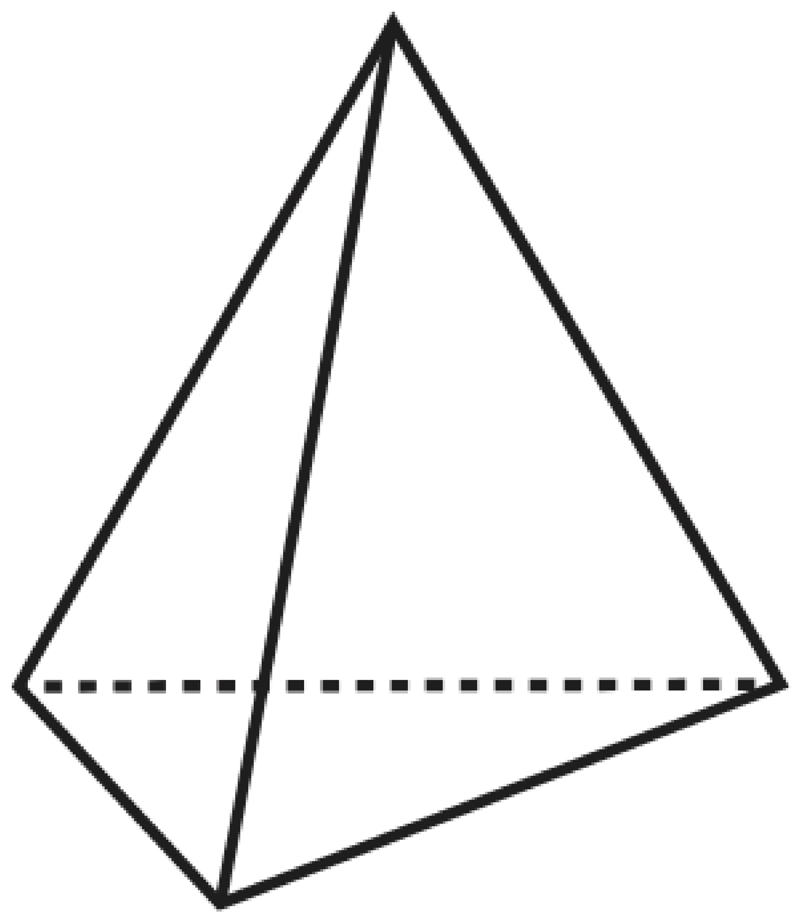
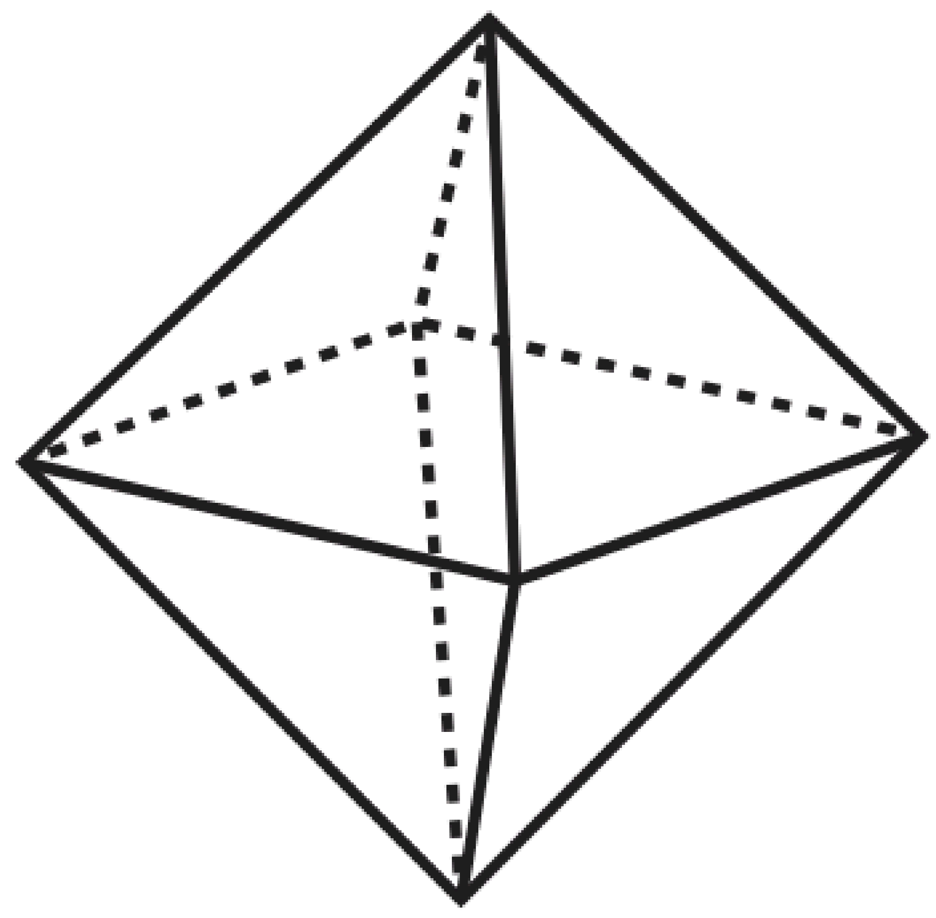

On the axes below sketch the graph of the gradient function $y = p'(x)$.




(iii) Is it true to say that the gradient of the function $p(x)$ is zero when $x = 0$?

Justify your response using the graphs on pages 8 and 9, and/or mathematical reasoning.

(c) A calculus class is exploring 3-D geometrical art. The students decide that they want to create wireframe models of a tetrahedron and an octahedron, similar to those shown below.

| Tetrahedron | Octahedron |
|---|---|
|  |  |
| Surface area $= \sqrt{3}a^2$ where $a$ represents the length of each edge of the tetrahedron | Surface area $= 2\sqrt{3}b^2$ where $b$ represents the length of each edge of the octahedron |

They will make each edge with a piece of wire, and join each piece together, and then cover each surface with paper to create lanterns

They wish to create each model so that the total resulting surface area of the two shapes is a minimum.

They have a total of 180 cm of wire to cut and use for both models.

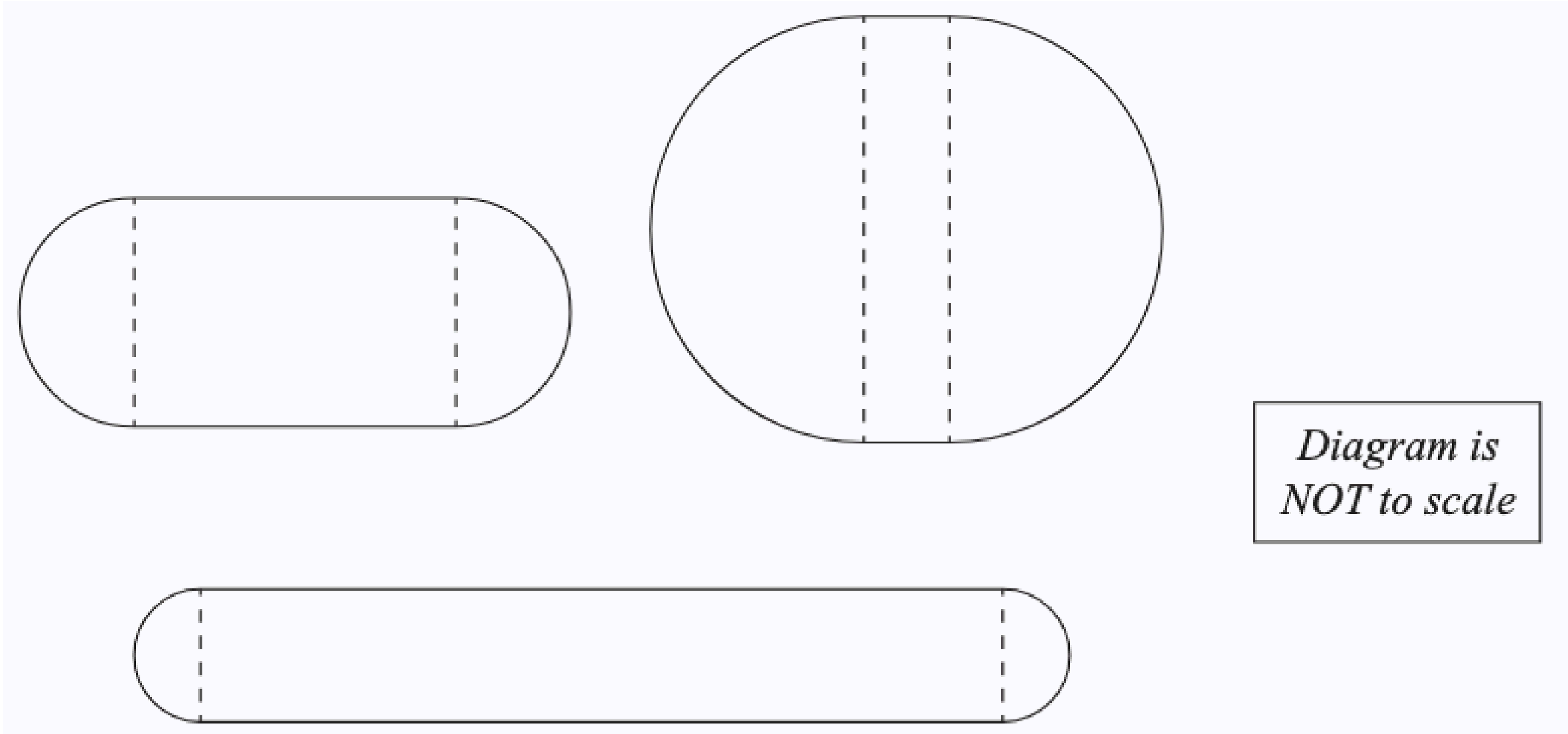
Determine the lengths of the edges, $a$ and $b$, required to minimise the total surface area of the two shapes when all of the wire is used.

Use calculus to justify that the surface area is a minimum.

(c) Use calculus to find the values of $x$ for which the graph of the function $f(x) = \frac{x^4}{4} - \frac{2x^3}{3} - 12x^2 + 10$ is increasing.

(d) A school is marking out a 400 m running track on its field. They want to be able to use the infield area (the rectangle between the semi-circles) to run PE games and activities. Due to this, they wish to maximise the rectangular area enclosed by the track. The track must consist of two straight sections and two semi-circular sections, and must be 400 m.

Three examples of possible field configurations are shown below:




Find the dimensions of the rectangle that maximise its area.

Use calculus methods to show that this is a maximum.

Circumference of a circle: $C = 2\pi r$

(c) A new car is travelling at 27.78 m $s^{-1}$ (100 km/hr), when the driver sees an obstruction in the road ahead. The driver applies the brakes, and decelerates at a rate of 2.5 m $s^{-2}$.

(i) How long will it take the car to come to a complete stop?

(ii) Using calculus methods, show that the distance the car travels between the time the driver applies the brakes, and the car coming to a complete stop, is approximately 154 m.

Continued

(iii) A car with new brake pads is capable of deceleration of 2.5 m $s^{-2}$. However, as a car's brakes age, they become less effective at slowing down the car.

If an older car's brakes were only able to allow for a deceleration of 2.1 m $s^{-2}$, what is the highest speed in which the car should be travelling so that it is able to come to a complete stop in the same distance (approx. 154 m) as the new car described in part (ii).

(b) The number of followers of a new social media account can be modelled by:

$$F = 5t^3 - 840t^2 + 42180t \qquad (0 \leq t \leq 90)$$

Where $t$ is days after the account is created, and $F$ is the number of followers.

(i) What is the rate of increase of followers on day 5?

(ii) What is the maximum number of followers the social media account has during the first 90 days?

(iii) How many days during the first 90 days does the social media account lose followers?

Use calculus methods to justify your answer.

(c) The curve $f(x) = x^3 - px^2 + 21x - 7$ has a tangent at $x = 4$ that also crosses the curve at $(0,-7)$.

Find the value of $p$ that makes this true.

(c) A curve is given by $y = 3x^3 - 9x^2 - 27x + 4$.

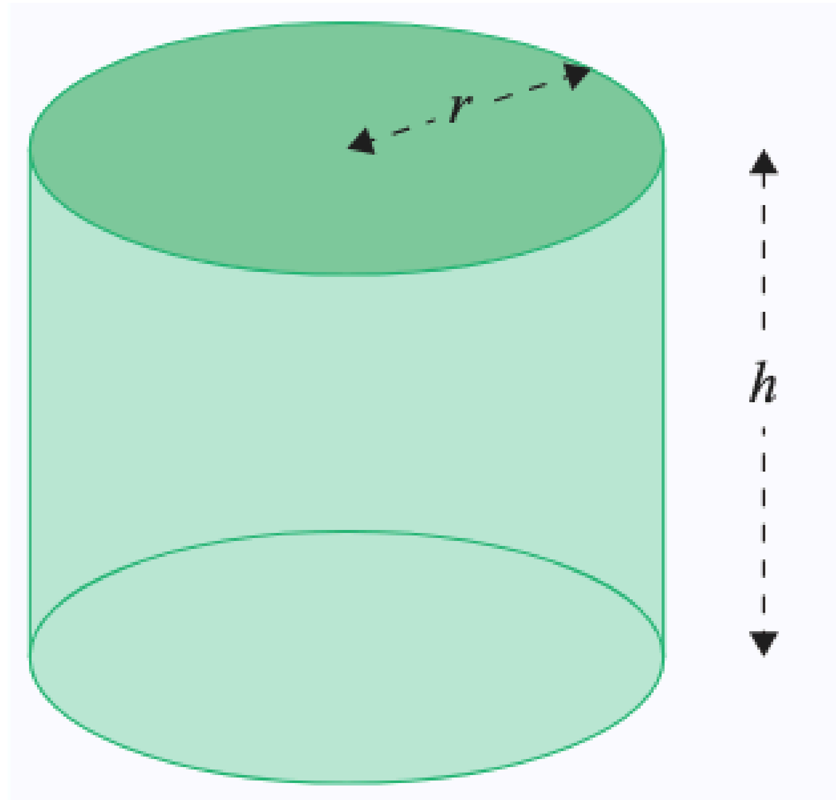
Using calculus methods:

(i) Find the $x$-coordinate of the local minimum.

(ii) Explain how you know this is a minimum point.

(d) A drink manufacturer would like to start a new line of cylindrical cups that are designed to keep hot liquids warm for as long as possible. To do this, they wish to minimise the surface area of the new cup, which includes a lid.

They would like the cup to hold a volume of 500 ml, and be cylindrical in shape.




Calculate the dimensions of the cylinder that would satisfy the above conditions.

Volume of a cylinder: $V = \pi r^2 h$

Surface area of a cylinder: $SA = 2\pi r^2 + 2\pi rh$

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(c) A microbiologist is growing bacteria in a Petri dish in a lab, and monitoring the population of bacteria in the dish.

After a certain number of days the bacteria colony will have used up all the nutrients in the Petri dish. At this time, the population will be at a maximum and it will then begin to decrease. When the microbiologist adds more nutrients to the Petri dish, the population of bacteria will begin to increase again.

The microbiologist thinks the population could be modelled by this equation:

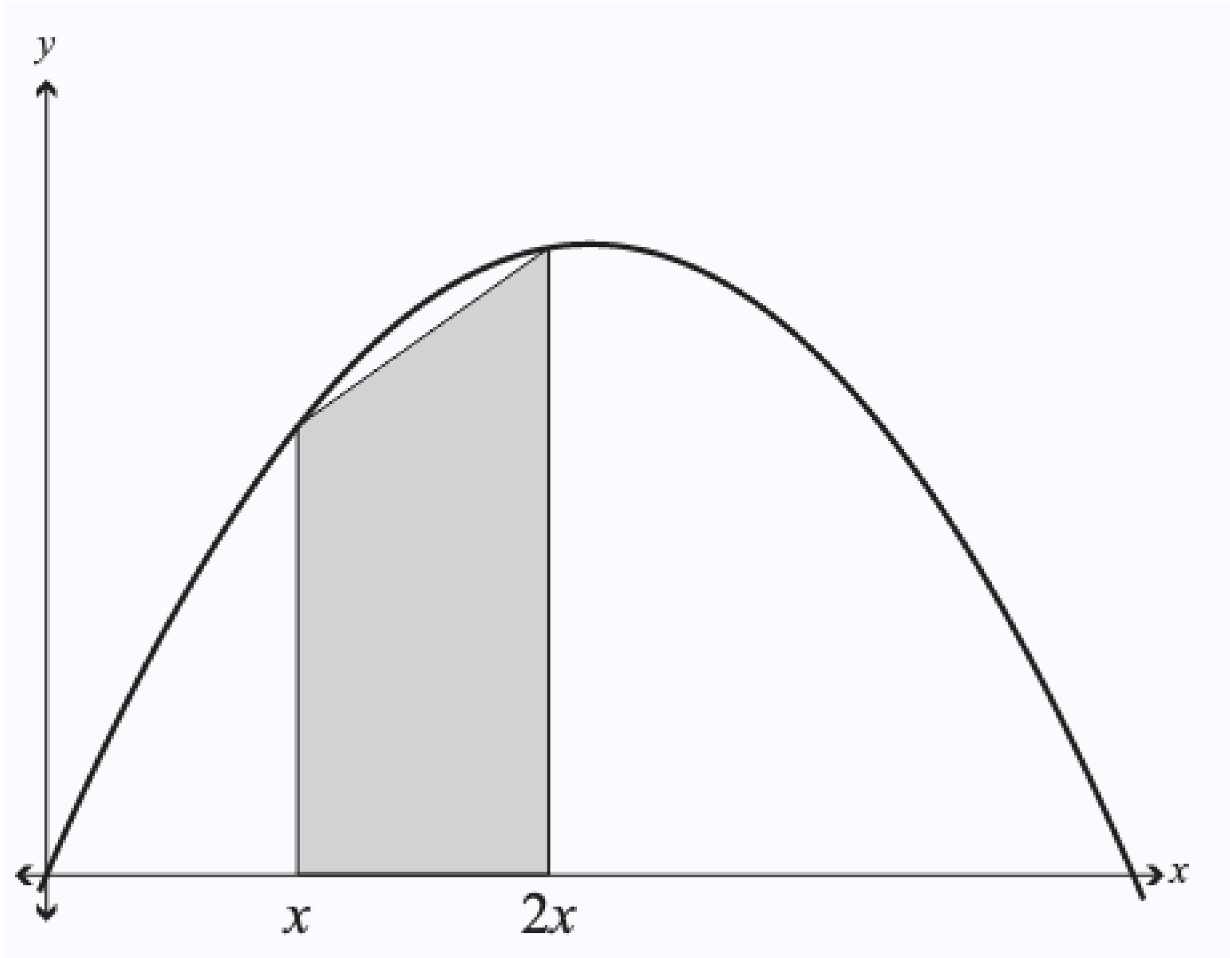
$$P(t) = t^3 - 60t^2 + 768t + 40\,960$$

where $P$ is the population of the bacteria, and $t$ is the number of days since the sample was prepared.

(i) What is the rate at which the population is decreasing on day 10?

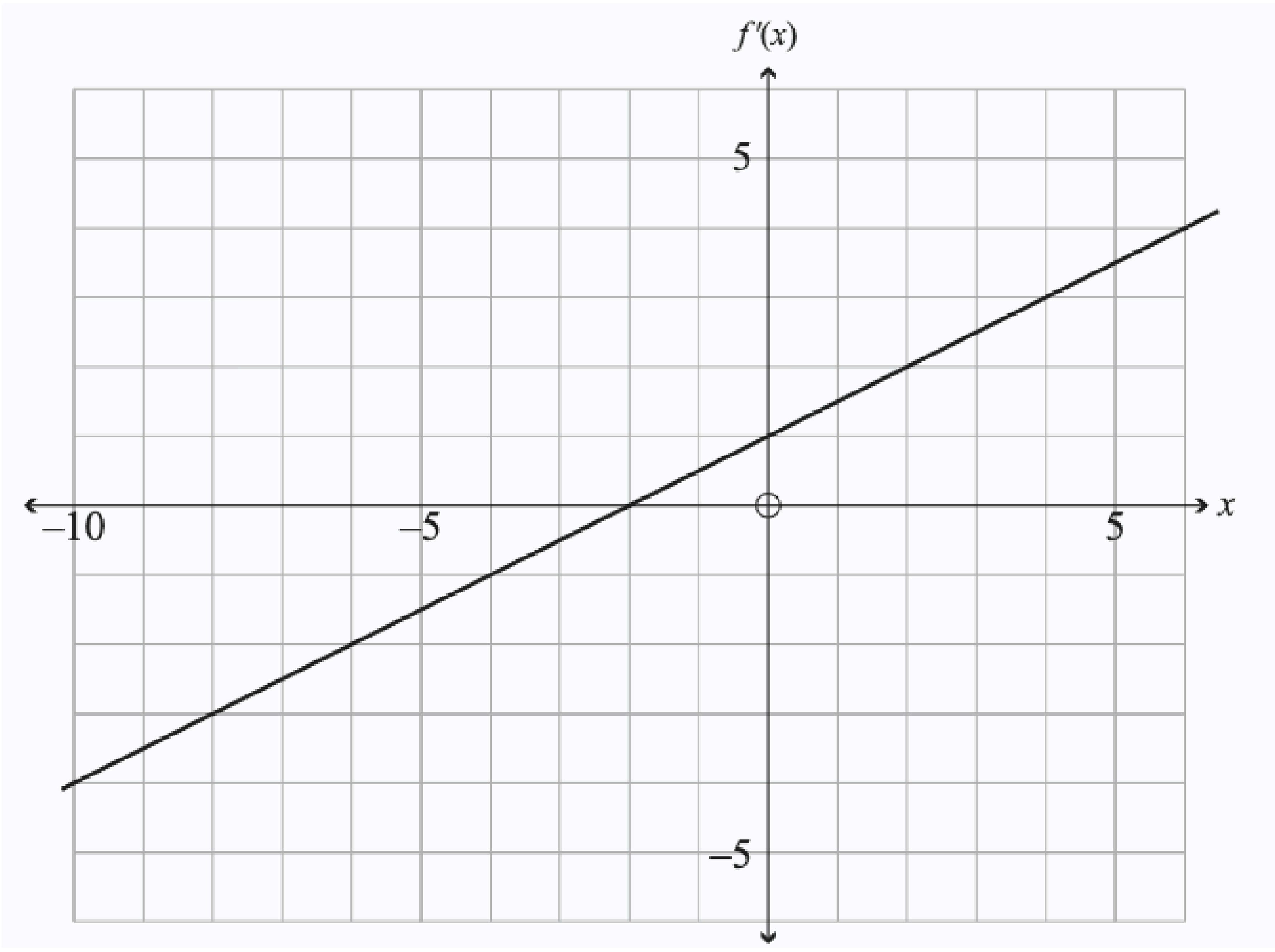
(ii) Use calculus methods to determine the **difference** in bacteria population size from the time all the nutrients in the Petri dish had been used up, to the time the microbiologist added more nutrients.

(d) A trapezium is drawn within a parabola given by $y = 8x - x^2$ and the $x$-axis, such that the two parallel sides are positioned at $x$ and $2x$, as shown in the graph below, where $0 \leq x \leq 4$.



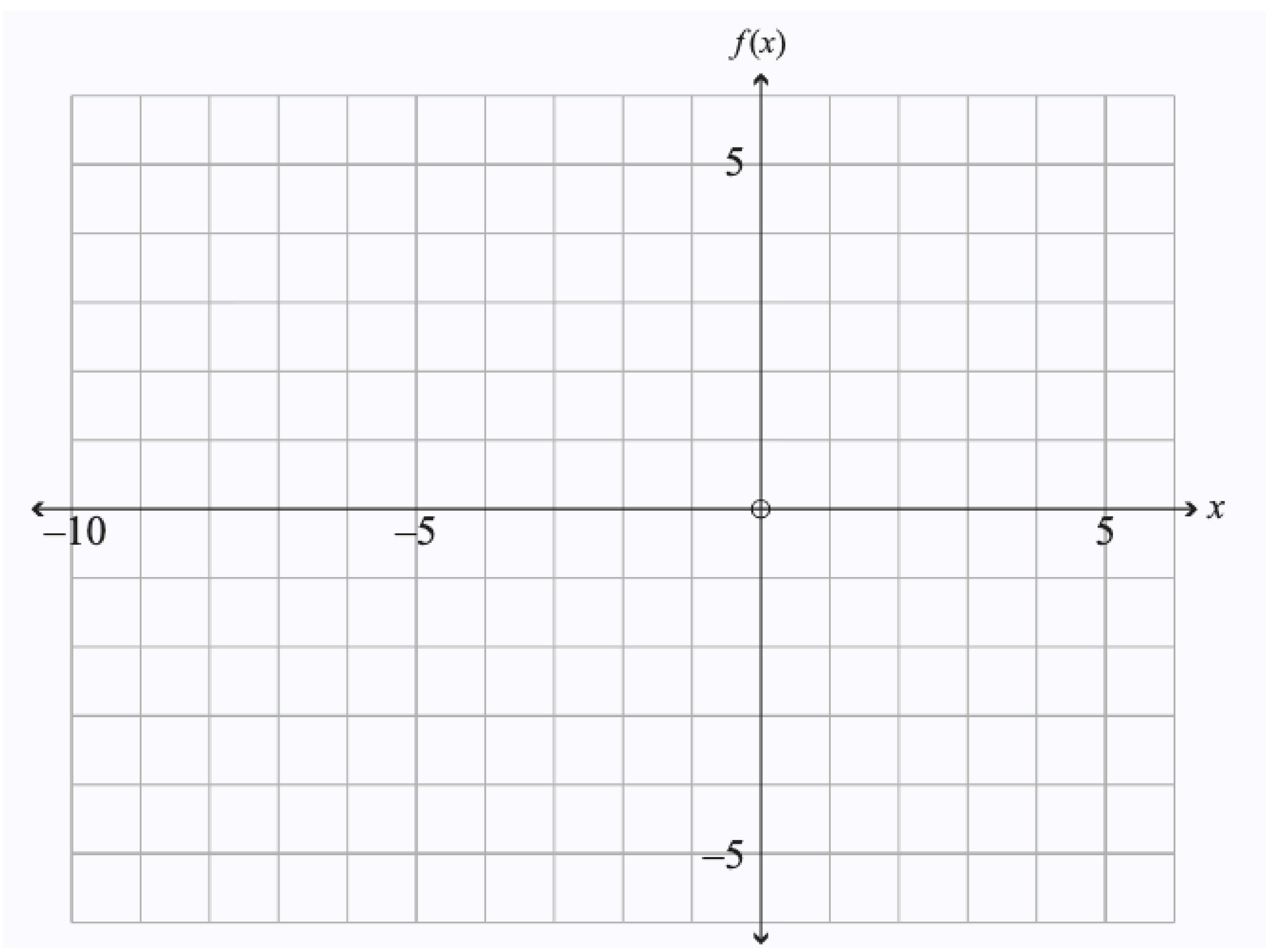

Given that the area of a trapezium is given by $A = \frac{1}{2}(a+b)h$, where $a$ and $b$ are the lengths of the parallel sides, and $h$ is the perpendicular distance between them, use calculus to find the maximum area of the trapezium.

(c) Shown on the graph below is a gradient function $f'(x)$:




(i) Find the equation for $f(x)$ which passes through (1,0).

(ii) Draw the graph of $f(x)$ on the axis below, labelling the coordinates of any intercept or stationary point(s).



(d) If $y = k - x$, where $k$ is a number, use calculus to show that the minimum value of $x^2 + 2y^2$ is equal to $\frac{2k^2}{3}$.

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(d) A cruise ship, the *Helena*, moves at a constant speed of 5 m $s^{-1}$.

A passenger who missed the ship gets onto a smaller boat. The smaller boat accelerates at 0.5 m $s^{-2}$ to catch up to the *Helena*.

When the smaller boat leaves the dock, the *Helena* is already 200 m away from the dock.

Find the distance from the dock at which the smaller boat catches up to the *Helena*.

(c) Hemi is part of a student research project on how caffeine enters the bloodstream. A new device measures caffeine levels in his blood.

He is monitored for 180 minutes (3 hours) and, at some point, he is given a cup of coffee to drink.

His caffeine levels can be modelled using this equation:

$$C(t) = \frac{t^3}{150} - 1.4t^2 + 80t + 240 \qquad \{0 \le t \le 180\}$$

where $C$ is the concentration (μg/L), and $t$ is time (minutes).

(i) Show that the rate of change of concentration is –16 μg/L/min after he has been there for 60 minutes.

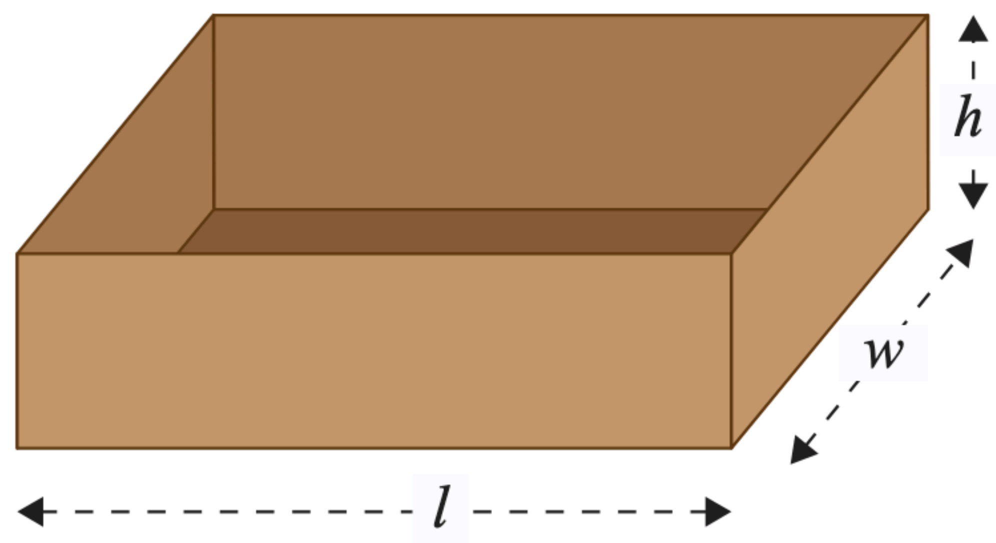
(ii) Use calculus to justify when Hemi was given the coffee, assuming the caffeine enters the bloodstream as soon as it is consumed, and continues to increase for some time after consumption.

(d) A function $f(x)$ is given by $f(x) = ax^3 + bx^2 + ax + 2$, where $a$ and $b$ are constants. The graph of $f(x)$ intersects the $x$-axis at $x = 1$ and has a stationary point at $x = -1$.

(i) Use calculus to find the $x$-coordinate of the other stationary point.

(ii) Use calculus methods to determine what type of stationary point this is.

(c) Evie is designing a lidless, rectangular cuboid container for her fishing trip, such as the one below.




The container is to be made of a maximum of 4.32 m² of material. She wants the box's width to be twice its height, and aims to maximise its volume.

Use calculus to:

- find the maximum volume of the box, given the limitations above
- prove that the volume is a maximum using calculus.

(d) A function $f(x)$ is defined as:

$$f(x) = \frac{x^4}{4} + \frac{(k-3)x^3}{3} - \frac{3kx^2}{2} + k$$

where $k$ is a positive constant.

Using calculus, determine the regions where the function $f(x)$ is decreasing.