91577 ALL EXCELLENCE QUESTIONS

#L3CALC

2013 Q1e:

If $u = 6 + ki$ and $v = 4 + ki$, find k if:

$$\arg(uv) = \frac{\pi}{4}$$

2013 Q2e:

Describe fully the locus of the points representing z if the following is purely imaginary:

$$\frac{z + 2i}{z - 2i}$$

2013 Q3e:

Find the values of k for which the following equation has no real roots:

$$6 + x - 4\sqrt{3x + k} = 0$$

2014 Q1e:

Find the equation of the locus described by:

$$|z - 1 + 2i| = |z + 1|$$

2014 Q2e:

The complex number z is given by the following equation, where p and q are real and $p > q > 0$:

$$z = \frac{1 + 3i}{p + qi}$$

Show that $p - 2q = 0$, given that:

$$\arg(z) = \frac{\pi}{4}$$

2014 Q3e:

α, β, and γ are the three roots of the cubic equation $ax^3 + bx^2 + cx + d = 0$, where a, b, c, and d are real numbers.

(i) Prove that:

$$\alpha + \beta + \gamma = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{-d}{a}$$

(ii) Hence prove that:

$$\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2 = \frac{bd}{a^2}$$

2015 Q1e:

If $x^2 + bx + c$ and $x^2 + dx + e$ have a common factor of $(x - p)$, prove the following equation, where b, c, d, e, and p are all real:

$$\frac{e - c}{b - d} = p$$

2015 Q2e:

Find the cartesian equation of the locus described by:

$$\arg\left(\frac{z - 2}{z + 5}\right) = \frac{\pi}{4}$$

2015 Q3e:

(i) Find each of the roots of the equation:

$$z^5 - 1 = 0$$

(ii) Let p be the root in part (i) with the smallest positive argument. Show that the roots in part (i) can be written as p, p^2, p^3, p^4.

2016 Q1e:

Find the gradient of the straight line of the locus:

$$|z - 2 + 3i| = |z - 1|$$

2016 Q2e:

Find the value of k if the following equation has equal roots:

$$8 - x + 2\sqrt{2x + k} = 0$$

2016 Q3e:

u and v are two complex numbers, such that:

$$|u + v|^2 = |u - v|^2$$

Prove that $u\overline{v}$ is purely imaginary.

2017 Q1e:

Find the Cartesian equation of the locus described by:

$$|z + 2 - 7i| = 2|z - 10 + 2i|$$

Write your answer in the form:

$$(x + A)^2 + (y + B)^2 = K$$

2017 Q2e:

Find all possible values of k that make the following equation a purely real number:

$$u = \frac{k + 4i}{1 + ki}$$

2017 Q3e:

z is a complex number where a and b are real numbers, such that:

$$z = \frac{a + bi}{a - bi}$$

Prove that:

$$\frac{z^2+1}{2z}=\frac{a^2-b^2}{a^2+b^2}$$

2018 Q1e:

Prove that $c^2+d^2=1$, if:

$$z=a+bi$$

$$\frac{z}{\bar{z}}=c+di$$

2018 Q2e:

The complex number $u=3+mi$ is on the locus of points defined by:

$$|z-8|=|z-4+2i|$$

Find the value of m.

2018 Q3e:

Solve the following equation for x in terms of k:

$$\frac{\sqrt{x+k}+\sqrt{x-k}}{\sqrt{x+k}-\sqrt{x-k}}=4$$

2019 Q1e:

Find the values of x and y, given that x and y are real, and:

$$\frac{1}{x+iy}-\frac{1}{1+i}=1-2i$$

2019 Q2e:

If $|z|=1$, and $z\neq 1$, prove that the following equation is purely imaginary:

$$\frac{1+z}{1-z}$$

2019 Q3e:

Prove that the following quadratic equation will have two distinct real solutions for all real values of k:

$$x^2+3kx+k^2=7x+3k$$

2020 Q1e:

Given the following equation where a and b are real constants:

$$T=\frac{a-bi}{a+bi}$$

Prove that:

$$\frac{1+T^2}{2T}=\frac{a^2-b^2}{a^2+b^2}$$

2020 Q2e:

Find the Cartesian equation of the locus, in the form $x^2+y^2=k$, described by:

$$|z+i|^2+|z-i|^2=10$$

2020 Q3e:

For complex numbers u and v, prove that the following is purely imaginary:

$$\frac{u}{v}$$

If:

$$|u+v| = |u-v|$$

2021 Q1e:

Given that the real part of the following equation is zero, and $z \neq_4$, prove that the locus of points described by z is given by the Cartesian equation $(x-2)^2 + (y-1)^2 = 5$:

$$\frac{z-2i}{z-4}$$

2021 Q2e:

Find the value of $|z|$ if z is a complex number and if:

$$|z+16| = 4|z+1|$$

2021 Q3e:

Solve the following equation:

$$z^2 = i(|z|^2 - 4)$$

2022 Q1e:

For complex numbers w and z, prove that:

$$|w+z|^2 - |w-\overline{z}| = 4\mathrm{Re}(w)\mathrm{Re}(z)$$

2022 Q2e:

Find the Cartesian equation of the locus, in the form $(x-a)^2 + (y-b)^2 = k^2$ described by the following equation:

$$|z+i| = 2|z-5i|$$

2022 Q3e:

Find the values of a and b if $z = a + bi$ is a non-zero complex number, and:

$$\frac{i}{z} + \frac{3}{\overline{z}} = 1$$

2023 Q1e:

The complex numbers u and v are $u = 3 + i$ and $v = 1 + 2i$. Determine the possible value(s) of the real constant k if:

$$\left|\frac{u}{v} + k\right| = \sqrt{k+2}$$

2023 Q2e:

Find the value of m, and the Cartesian equation of the locus, where the straight line with equation $y = mx - 1$ (m is a real constant and $m > 0$) is a tangent to the locus described by:

$$|z - 2 + i| = \sqrt{3}$$

2023 Q3e:

Find the value of k, if $z+2i=iz+k$ $(k \subset \text{Real})$, $\text{Im}(w)=8$, and:

$$\frac{w}{z}=2+2i$$

2024 Q1e:

Consider the complex numbers $u=2+3ki$ and $v=4+5ki$, where k is a real constant. Show that the following complex number will not lie on the line $y=x$ in the Argand diagram for any value of k:

$$w=\frac{u}{v}$$

2024 Q2e:

The locus of a complex number z in the form $(x-a)^2+(y-b)^2=k$, is described by:

$$|z-1-7i|=2|z-4-4i|$$

The complex number $u=3+di$ lies on this locus. Find the Cartesian equation of the locus of z, and also find the complex number(s) u.

2024 Q3e:

Given the following where p is a real constant:

$$x+\frac{1}{x}=p$$

Find the value of the following equation in terms of p:

$$x^3+\frac{1}{x^3}$$

2025 Q1e:

Find the Cartesian equation of the locus of the complex number z, described by the following equation in the form $ay^2-bx^2=k$:

$$|z-5i|-|z+5i|=4$$

2025 Q2e:

Find the possible value(s) of d (d is a real constant), given that:

$$\arg\left(\frac{d+6i}{1-di}\right)=\frac{\pi}{4}$$

2025 Q3e:

Given that u and v are complex numbers, solve the following system, (giving answers in the form $a+bi$):

$$\begin{cases} ui+2v=3 \\ u+(1-i)v=4 \end{cases}$$