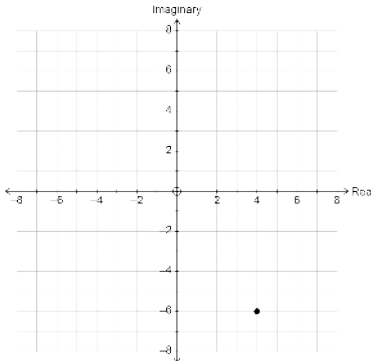


Assessment Schedule – 2021**Calculus: Apply the algebra of complex numbers in solving problems (91577)****Evidence Statement**

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
ONE (a)	$(d + 5i)(3 - 4i) = 38 - 9i$ $3d - 4di + 15i - 20i^2 = 38 - 9i$ $3d + 20 + (15 - 4d)i = 38 - 9i$ $d = 6$	Correct solution.		
(b)	$z = \frac{26(2 - 3i)}{(2 + 3i)(2 - 3i)}$ $= \frac{26(2 - 3i)}{13}$ $= 4 - 6i$ 	Correct solution correctly plotted on Argand diagram.		
(c)	$f(-1) = f(2)$ $-1 + 3 - a + b = 8 + 12 + 2a + b$ $2 - 20 = 2a + a$ $a = -6$ $f(-2) = 0$ $-8 + 12 - 2a + b = 0$ $16 + b = 0$ $b = -16$	Correct value for a OR b .	Correct values for a AND b .	
(d)	$\arg\left(\frac{1 + 3i - 1}{1 + 3i - 2i}\right) = \arg\left(\frac{3i}{1 + i}\right)$ $= \arg\left(\frac{3i(1 - i)}{(1 + i)(1 - i)}\right)$ $= \arg\left(\frac{3i - 3i^2}{1 - i^2}\right)$ $= \arg\left(\frac{3 + 3i}{2}\right)$ $= \arg\left(\frac{3}{2} + \frac{3i}{2}\right)$ $= \frac{\pi}{4}$	Correct simplification to $\left(\frac{3i}{1 + i}\right)$.	Correct solution.	

(e)	$\frac{z-2i}{z-4} = \frac{x+yi-2i}{x+yi-4}$ $= \frac{x+(y-2)i}{(x-4)+yi}$ $= \frac{(x+(y-2)i)((x-4)-yi)}{((x-4)+yi)((x-4)-yi)}$ $= \frac{x(x-4)-xyi+(y-2)(x-4)i-y(y-2)i^2}{(x-4)^2-y^2i^2}$ $= \frac{x(x-4)-xyi+(y-2)(x-4)i+y(y-2)}{(x-4)^2+y^2}$ Real part = 0 $\Rightarrow x(x-4)+y(y-2)=0$ $x^2-4x+y^2-2y=0$ $(x-2)^2-4+(y-1)^2-1=0$ $(x-2)^2+(y-1)^2=5$	Line 3.	Line 5.	Correct solution.
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NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
TWO (a)	$u = 2\text{cis}\left(\frac{\pi}{2}\right)$ $z = \frac{u}{w} = \frac{2\text{cis}\frac{\pi}{2}}{2\text{cis}\frac{2\pi}{3}}$ $= \text{cis}\left(\frac{\pi}{2} - \frac{2\pi}{3}\right)$ $= \text{cis}\left(-\frac{\pi}{6}\right)$ $= \cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)$ $= \frac{\sqrt{3}}{2} - 0.5i \text{ or } 0.866 - 0.5i$	Correct solution. Accept answer in rectangular or polar form.		
(b)	$x^2 - 12qx + 20q^2 = 0$ $(x - 6q)^2 - 36q^2 + 20q^2 = 0$ $(x - 6q)^2 = 16q^2$ $x - 6q = \pm 4q$ $x = 6q \pm 4q$ $x = 10q \text{ or } 2q$	Correct solution.		
(c)	$\frac{(a + bi)(b + ai)}{(b - ai)(b + ai)}$ $= \frac{ab + a^2i + b^2i + abi^2}{b^2 + abi - abi - a^2i^2}$ $= \frac{ab + (a^2 + b^2)i - ab}{b^2 + a^2}$ $= \frac{(a^2 + b^2)i}{b^2 + a^2}$ $= i$	Correct 2nd line.	Correct solution.	
(d)	$z^3 = k^6 + k^6i$ $= \sqrt{2}k^6\text{cis}\left(\frac{\pi}{4}\right)$ $z_1 = \left(\sqrt{2}\right)^{\frac{1}{3}}k^2\text{cis}\left(\frac{\pi}{12}\right) = 2^{\frac{1}{6}}k^2\text{cis}\left(\frac{\pi}{12}\right)$ $z_2 = \left(\sqrt{2}\right)^{\frac{1}{3}}k^2\text{cis}\left(\frac{3\pi}{4}\right) = 2^{\frac{1}{6}}k^2\text{cis}\left(\frac{3\pi}{4}\right)$ $z_3 = \left(\sqrt{2}\right)^{\frac{1}{3}}k^2\text{cis}\left(\frac{-7\pi}{12}\right) = 2^{\frac{1}{6}}k^2\text{cis}\left(\frac{-7\pi}{12}\right)$	One correct solution. $2^{\frac{1}{6}} = 1.122$	Three correct solutions. Accept $\frac{17\pi}{12}$.	

(e)	$ x + iy + 16 = 4 x + iy + 1 $ $ (x + 16) + iy = 4 (x + 1) + iy $ $\sqrt{x^2 + 32x + 256 + y^2} = 4\sqrt{x^2 + 2x + 1 + y^2}$ $x^2 + 32x + 256 + y^2 = 16(x^2 + 2x + 1 + y^2)$ $x^2 + 32x + 256 + y^2 = 16x^2 + 32x + 16 + 16y^2$ $240 = 15x^2 + 15y^2$ $16 = x^2 + y^2$ $4 = \sqrt{x^2 + y^2}$ $\therefore z = 4$	3rd line.	5th line.	Correct solution.
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NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
THREE (a)	$\sqrt{11}$	Correct solution.		
(b)	$\frac{18}{(4-2\sqrt{3})} \times \frac{(4+2\sqrt{3})}{(4+2\sqrt{3})} = \frac{72+36\sqrt{3}}{16-12}$ $= 18+9\sqrt{3}$	Correct solution.		
(c)	$z_1 = 2+5i$ $z_2 = 2-5i$ $(z-2-5i)(z-2+5i) = z^2 - 2z + 5zi - 2z + 4 - 10i - 5iz + 10i - 25i^2$ $= z^2 - 4z + 4 + 25$ $= z^2 - 4z + 29$ $f(z) = (z^2 - 4z + 29)(pz + q)$ $p = 4$ Coeff of z^2 : $(z^2 - 4z + 29)(4z + q) = -19$ $q - 16 = -19$ $q = -3$ $f(z) = (z^2 - 4z + 29)(4z - 3)$ $z_3 = \frac{3}{4}$ $A = -87$	The other two solutions found. OR A found.	The other two solutions found. AND A found.	
(d)	$6\sqrt{2x} - 5 = 6\sqrt{2x+m}$ $(6\sqrt{2x} - 5)^2 = (6\sqrt{2x+m})^2$ $36 \times 2x - 60\sqrt{2x} + 25 = 36(2x+m)$ $72x - 60\sqrt{2x} + 25 = 72x + 36m$ $-60\sqrt{2x} = 36m - 25$ $\sqrt{2x} = \frac{25-36m}{60}$ $2x = \left(\frac{25-36m}{60}\right)^2$ $x = \frac{(25-36m)^2}{7200}$	Correct 3rd line.	Correct solution.	

(e)	$z^2 = i(z ^2 - 4)$ $(x + iy)^2 = i(x^2 + y^2 - 4)$ $x^2 + 2xyi + y^2i^2 = (x^2 + y^2 - 4)i$ $x^2 - y^2 + 2xyi = (x^2 + y^2 - 4)i$ Real component: $x^2 - y^2 = 0$ $x^2 = y^2$ $x = y$ or $x = -y$ Imaginary component: $x^2 + y^2 - 4 = 2xy$ If $x = y$, then $2x^2 - 4 = 2x^2$ impossible If $x = -y$, then $2x^2 - 4 = -2x^2$ $4x^2 = 4$ $x = \pm 1$ $x = 1 \Rightarrow y = -1 \Rightarrow z = 1 - i$ $x = -1 \Rightarrow y = 1 \Rightarrow z = -1 + i$	Finding $x^2 = y^2$.	Identifying $x = -y$ is only possibility.	Correct solution.
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NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 7	8 – 14	15 – 19	20 – 24