

Assessment Schedule – 2022**Calculus: Apply the algebra of complex numbers in solving problems (91577)****Evidence Statement**

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
ONE (a)	$\frac{12k}{1+\sqrt{5}} \times \frac{1-\sqrt{5}}{1-\sqrt{5}} = \frac{12k-12k\sqrt{5}}{-4}$ $= -3k + 3k\sqrt{5}$	Correct solution.		
(b)	$\frac{u}{v} = m^3 \operatorname{cis}\left(\frac{\pi}{3} - \frac{\pi}{5}\right)$ $= m^3 \operatorname{cis}\left(\frac{2\pi}{15}\right)$	Correct solution.		
(c)	$uv = (3+2i)(4+2i)$ $= 8+14i$ $uvw = (8+14i)(2+ki)$ $= (16-14k) + (8k+28)i$ $\operatorname{Arg}(uvw) = \frac{\pi}{4} \text{ means } \operatorname{Re} = \operatorname{Im}$ $(16-14k) = (8k+28)$ $-12 = 22k$ $k = \frac{-12}{22} = \frac{-6}{11}$	$(16-14k) + (8k+28)i$	Correct solution.	
(d)	$x - 2\sqrt{x+p} = -5$ $x+5 = 2\sqrt{x+p}$ $x^2 + 10x + 25 = 4(x+p)$ $x^2 + 6x + 25 - 4p = 0$ $b^2 - 4ac = 0$ $36 - 4(25 - 4p) = 0$ $36 - 100 + 16p = 0$ $16p = 64$ $p = 4$	Correct quadratic.	Correct solution.	
(e)	$ w+z ^2 - w-\bar{z} ^2 = 4\operatorname{Re}(w)\operatorname{Re}(z)$ $\text{Let } w = a+bi \text{ and } z = c+di$ $w+z = a+bi+c+di$ $= a+c+(b+d)i$ $w-\bar{z} = a+bi-(c-d)i$ $= a-c+(b+d)i$ $ w+z ^2 - w-\bar{z} ^2 = (a+c)^2 + (b+d)^2 - ((a-c)^2 + (b+d)^2)$ $= a^2 + 2ac + c^2 + b^2 + 2bd + d^2 - (a^2 - 2ac + c^2 + b^2 + 2bd + d^2)$ $= 4ac$ $= 4\operatorname{Re}(w)\operatorname{Re}(z)$		Correct expanded form for $ w+z ^2 - w-\bar{z} ^2$	Correct solution.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
TWO (a)	$(-2)^3 - 3(-2)^2 - 2b + 9 = 3$ $-8 - 12 - 2b + 9 = 3$ $-2b = 14$ $b = -7$	Correct solution.		
(b)	$\text{Let } z = a + bi$ $a + bi + 4(a - bi) = 15 + 12i$ $5a - 3bi = 15 + 12i$ $a = 3, b = -4$ $z = 3 - 4i$	Correct solution.		
(c)	$\frac{z^2 - 6z + 45}{z + 4\sqrt{z^3 - 2z^2 + hz + 180}}$ $\frac{z^3 + 4z^2}{-6z^2 + hz}$ $\frac{-6z^2 - 24z}{(h + 24)z + 180}$ $\frac{45z + 180}{0}$ $h + 24 = 45$ $h = 21$ $z^2 - 6z + 45 = 0$ $(z - 3)^2 + 36 = 0$ $(z - 3)^2 = -36$ $z - 3 = \pm 6i$ $z = 3 \pm 6i$	Correct value of h . OR Other solutions found correctly.	Correct value of h . AND Other solutions found algebraically.	
(d)	$w = \frac{4}{z} - 2 \quad z = 1 - \sqrt{3}i$ $w = \frac{4}{1 - \sqrt{3}i} - 2$ $= \frac{4}{1 - \sqrt{3}i} \times \frac{1 + \sqrt{3}i}{1 + \sqrt{3}i} - 2$ $= \frac{4 + 4\sqrt{3}i}{4} - 2$ $= 1 + \sqrt{3}i - 2$ $= -1 + \sqrt{3}i$ $\arg(w) = \frac{2\pi}{3} \text{ or } \arg(w) = 120^\circ$	Correct expression for w with rationalised denominator.	Correct solution.	

(e)	$ z + i = 2 z - 5i $ $ x + iy + i = 2 x + iy - 5i $ $ x + (y+1)i = 2 x + (y-5)i $ $\sqrt{x^2 + (y+1)^2} = 2\sqrt{x^2 + (y-5)^2}$ $x^2 + (y+1)^2 = 4(x^2 + (y-5)^2)$ $x^2 + y^2 + 2y + 1 = 4x^2 + 4y^2 - 40y + 100$ $0 = 3x^2 + 3y^2 - 42y + 99$ $x^2 + y^2 - 14y + 33 = 0$ $x^2 + (y-7)^2 - 16 = 0$ $x^2 + (y-7)^2 = 16$		Correct expanded quadratic.	Correct solution.
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	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
THREE (a)	$4 + 9i$ correctly plotted.	Correct solution.		
(b)	$z^2 + 6kz + 152k^2 = 0$ $(z + 3k)^2 + 6k^2 = 0$ $(z + 3k)^2 = -6k^2$ $z + 3k = \pm\sqrt{6}ki$ $z = -3k \pm \sqrt{6}ki$	Correct solution.		
(c)	$z^3 = -k^6i$ $= k^6 \operatorname{cis}\left(\frac{-\pi}{2}\right)$ Solution interval $= \frac{2\pi}{3}$ $z_1 = k^2 \operatorname{cis}\left(\frac{-\pi}{6}\right)$ $z_2 = k^2 \operatorname{cis}\left(\frac{3\pi}{6}\right) = k^2 \operatorname{cis}\left(\frac{\pi}{2}\right)$ $z_3 = k^2 \operatorname{cis}\left(\frac{7\pi}{6}\right) = k^2 \operatorname{cis}\left(\frac{-5\pi}{6}\right)$	One correct solution.	Three correct solutions.	
(d)	$ z - z = i$ $z = a + bi$ $\sqrt{a^2 + b^2} - (a + bi) = i$ $\sqrt{a^2 + b^2} - a - bi = i$ Equating imaginary parts: $-bi = i$ $b = -1$ Equating real parts: $\sqrt{a^2 + (-1)^2} - a = 0$ $\sqrt{a^2 + 1} = a$ $a^2 + 1 = a^2$ $1 = 0$ Not possible.	Correct value of b .	Correct solution.	

(e)	$\frac{i}{z} + \frac{3}{\bar{z}} = 1$ $\frac{i}{a+bi} + \frac{3}{a-bi} = 1$ $\frac{i(a-bi) + 3(a+bi)}{(a+bi)(a-bi)} = 1$ $\frac{ai + b + 3a + 3bi}{a^2 + b^2} = 1$ $\frac{b+3a}{a^2+b^2} + \frac{(a+3b)i}{a^2+b^2} = 1$ Equating imaginary parts: $a+3b=0 \text{ or } a=-3b$ Equating real parts: $\frac{b+3a}{a^2+b^2} = 1$ $b+3a = a^2 + b^2$ $-8b = 10b^2$ $2b(5b+4) = 0$ $b=0 \text{ or } b = -\frac{4}{5}$ If $b=0$, then $a=-3b=0$ not allowed $b = -\frac{4}{5}$ $a = -3b = \frac{12}{5}$ $\left(z = \frac{12}{5} - \frac{4}{5}i \right)$		$a = -3b.$	Correct solution.
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Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 6	7 – 14	15 – 20	21 – 24