

Assessment Schedule – 2023**Calculus: Apply the algebra of complex numbers in solving problems (91577)****Evidence Statement**

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
ONE (a)	$(5 - 2\sqrt{p})(5 - 2\sqrt{p}) = 25 - 10\sqrt{p} - 10\sqrt{p} + 4p$ $= 25 + 4p - 20\sqrt{p}$	Required expression.		
(b)	No real roots so $b^2 - 4ac < 0$ $16 - 4 \times 4 \times (3r - 2) < 0$ $16 - 48r + 32 < 0$ $-48r + 48 < 0$ $-48r < -48$ $r > 1$	Correct inequality.		
(c)	$\frac{z}{w} = \frac{p + qi}{a + bi} = \frac{(p + qi)(a - bi)}{(a + bi)(a - bi)}$ $= \frac{ap - bpi + aqi - bqi^2}{a^2 - b^2i^2}$ $= \frac{ap - bpi + aqi + bq}{a^2 + b^2} \quad \#(1)$ $= \frac{ap + bq + (aq - bp)i}{a^2 + b^2}$ $\operatorname{Re}\left(\frac{z}{w}\right) = 0 \Rightarrow \frac{ap + bq}{a^2 + b^2} = 0$ $\Rightarrow ap + bq = 0$ $\Rightarrow ap = -bq$ As required.	Reaching stage #(1).	Proof completed.	
(d)	$z_1 = 5 - i$ so $z_2 = 5 + i$ $z - 5 = i \Rightarrow (z - 5)^2 = i^2$ $\Rightarrow z^2 - 10z + 26 = 0$ $f(z) = (Az + B)(z^2 - 10z + 26)$ $\Rightarrow A = 1; B = 2$ i.e. $f(z) = (z + 2)(z^2 - 10z + 26)$ So $z_3 = -2$ and $d = 52$	- The other two solutions found. - OR d found.	- The other two solutions found. - AND d found.	

(e)	$\left \frac{u}{v} + k \right = \sqrt{k+2}$ $\left \frac{3+i}{1+2i} + k \right = \sqrt{k+2}$ $\left \frac{(3+i)(1-2i)}{(1+2i)(1-2i)} + k \right = \sqrt{k+2}$ $\left \frac{5-5i}{5} + k \right = \sqrt{k+2}$ $ 1-i+k = \sqrt{k+2}$ $\sqrt{(1+k)^2 + 1} = \sqrt{k+2} \quad \text{\#(1)}$ $(1+k)^2 + 1 = k+2$ $k^2 + 2k + 2 = k+2$ $k^2 + k = 0 \quad k(k+1) = 0$ Either $k = 0$ or $k = -1$	Expressing $\frac{u}{v}$ in the form $1 - i$.	Reaching stage \#(1).	E7 Not including $k = 0$ as a valid solution. OR Correct solution but with one minor error. E8 Finding $k = 0$ and $k = -1$. Both solutions required, with valid and clear justification.
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NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
TWO (a)	$q^4 \operatorname{cis} \left(\frac{9\pi}{40} \right)$	Correct answer.		
(b)	$ z - w = (1 + ki) - (7 - ki) $ $= -6 + 2ki $ $= \sqrt{36 + 4k^2}$	Correct expression.		
(c)	$13z = (11 - 3i)(z + 1)$ $13z = 11z + 11 - 3iz - 3i$ $2z + 3iz = 11 - 3i$ $z(2 + 3i) = 11 - 3i$ $z = \frac{11 - 3i}{2 + 3i}$ #(1) $z = 1 - 3i$ $\operatorname{Arg} z = -71.6^\circ$ (or -1.25 rads) (or 288.4°)	Reaching stage #(1).	Correct value for $\operatorname{Arg} z$.	
(d)	$z^3 = -64m^{12} = 64m^{12} \operatorname{cis} \pi$ $\vartheta_1 = \frac{\pi}{3}$ $\vartheta_2 = \pi$ $\vartheta_3 = \frac{5\pi}{3} = -\frac{\pi}{3}$ $z_1 = 4m^4 \operatorname{cis} \frac{\pi}{3}$ $z_2 = 4m^4 \operatorname{cis} \pi$ $z_3 = 4m^4 \operatorname{cis} \frac{5\pi}{3} = 4m^4 \operatorname{cis} \left(-\frac{\pi}{3} \right)$ Or equivalent.	One correct solution.	All three correct solutions, with appropriate justification.	

(e)	Let $z = x + yi$, then $ x + yi - 2 + i = \sqrt{3}$ $ (x - 2) + (y + 1)i = \sqrt{3}$ $\sqrt{(x - 2)^2 + (y + 1)^2} = \sqrt{3} \quad \#(1)$ $(x - 2)^2 + (y + 1)^2 = 3$ Let $y = mx - 1$, then $(x - 2)^2 + (mx - 1 + 1)^2 = 3$ $x^2 - 4x + 4 + m^2x^2 = 3$ $(1 + m^2)x^2 - 4x + 1 = 0 \quad \#(2)$ Tangent gives $b^2 - 4ac = 0$ $16 - 4(1 + m^2) \times 1 = 0$ $16 - 4 - 4m^2 = 0$ $12 = 4m^2$ $m^2 = 3$ $m = \sqrt{3}$ Do not penalise $m = \pm\sqrt{3}$	• Reaching stage #(1).	• Reaching stage #(2) .	E 7 Correct solution but with one minor error. E 8 Correct value Of m with valid and clear justification.
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No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
THREE (a)	$f(-3) = 30$ $-54 + 9p - 21 - 3 = 30$ $9p - 78 = 30$ $9p = 108$ $p = 12$	Correct value of p.		
(b)	$\frac{n-i}{2-3i} = 3+4i$ $n-i = (3+4i)(2-3i)$ $n-i = 6-9i+8i+12$ $n-i = 18-i$ $n = 18$ Alternative method: $\frac{n-i}{2-3i} = 3+4i$ $\frac{(n-i)(2+3i)}{(2-3i)(2+3i)} = 3+4i$ $\frac{2n+3ni-2i+3}{4+9} = 3+4i$ $\frac{2n+3+(3n-2)i}{13} = 3+4i$ Comparing real parts gives: $\frac{2n+3}{13} = 3$, giving $n = 18$ OR Comparing imaginary parts gives: $\frac{3n-2}{13} = 4$, giving $n = 18$	Correct value of n.		
(c)	$16(4x-w) = (5-8\sqrt{x})(5-8\sqrt{x})$ $64x-16w = 25-80\sqrt{x}+64x$ #1) $80\sqrt{x} = 25+16w$ $\sqrt{x} = \frac{25+16w}{80}$ $x = \left(\frac{25+16w}{80}\right)^2$ Or equivalent.	Reaching stage #1).	Correct expression for x.	

(d)	$\frac{1}{x+yi} = 1 - \frac{1}{1+pi}$ $\frac{1}{x+yi} = \frac{1+pi-1}{1+pi}$ $\frac{1}{x+yi} = \frac{pi}{1+pi} \quad \#(1)$ $x+yi = \frac{1+pi}{pi}$ $x+yi = \frac{(1+pi)i}{pi \times i}$ $x+yi = \frac{i-p}{-p}$ $x+yi = 1 - \frac{1}{p}i$ So $x = 1$ and $y = -\frac{1}{p}$	Reaching stage #(1).	Correct values for x and y.	
(e)	$z+2i = iz+k$ $z-iz = k-2i$ $z(1-i) = k-2i$ $z = \frac{k-2i}{1-i} \quad \#(1)$ Then $w = z(2+2i)$ $w = \frac{k-2i}{1-i} \times (2+2i)$ $w = \frac{(k-2i)(2+2i)(1+i)}{(1-i)(1+i)}$ $w = \frac{(k-2i)(2+4i+2i^2)}{1-i^2}$ $w = \frac{(k-2i)4i}{2} \quad \#(2)$ $w = (k-2i)2i$ $w = 2ki - 4i^2$ $w = 4 + 2ki \quad \#(3)$ Therefore $\text{Im}(w) = 8 \Rightarrow 2k = 8$ $k = 4$	Reaching stage #(1).	Reaching stage #(2).	E 7 Reaching stage #(3) . OR Correct solution but with one minor error. E 8 Correct value of k, with valid and clear justification.

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No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 7	8 – 12	13 – 18	19 – 24