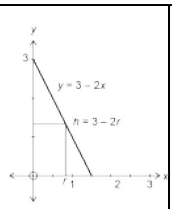
Assessment Schedule – 2021

Calculus: Apply differentiation methods in solving problems (91578)

Evidence Statement

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
ONE (a)	$\frac{dy}{dx} = 3e^{3x}\sin(2x) + e^{3x}\cos(2x).2$	Correct derivative.		
(b)(i) (ii)	(1) $x < 2, x = 4$ (2) $3 < x < 6$ 3	2 out of 3 correct responses.		
(c)	$y = (2x+3)e^{x^2}$ $\frac{dy}{dx} = 2e^{x^2} + (2x+3)(2x)e^{x^2}$ $\frac{dy}{dx} = 2e^{x^2}(1 + x(2x+3))$ $\frac{dy}{dx} = 2e^{x^2}(2x^2+3x+1)$ $\frac{dy}{dx} = 0$ for stationary points. $2e^{x^2} = 0$ has no solutions since $2e^{x^2} > 0$ $2x^2+3x+1=0$ $x = -\frac{1}{2}$ *or* $x = -1$	Correct derivative.	Correct solution with correct derivative.	
(d)	$x = t^2 + 3t$ $\frac{dx}{dt} = 2t+3$ $y = t^2\ln(2t-3)$ $\frac{dy}{dt} = 2t\ln(2t-3) + \frac{2t^2}{2t-3}$ $\frac{dy}{dx} = \frac{2t\ln(2t-3) + \frac{2t^2}{2t-3}}{2t+3}$ At $(10,0)$: $t^2+3t=10$ $t^2+3t-10=0$ $(t+5)(t-2)=0$ $t=-5$ or $t=2$ Since $t > \frac{3}{2}$, $t = 2$ $\frac{dy}{dx} = \frac{4\ln(1)+8}{7}$ $\frac{dy}{dx} = \frac{8}{7}$	$\frac{dy}{dt}$ correct.	$\frac{dy}{dt}$ correct And $t^2+3t=10$ solved to find $t=-5$ or $t=2$	T1: Correct solution with correct $\frac{dy}{dx}$.

(e)	$V = \pi r^2 h$ $= \pi r^2 (3 - 2r)$ $= 3\pi r^2 - 2\pi r^3$ $\frac{dV}{dr} = 6\pi r - 6\pi r^2$ At maximum, $\frac{dV}{dr} = 0$ $6\pi r(1-r) = 0$ $r = 0$ (no) $\therefore r = 1$ $V = \pi 1^2 (3 - 2 \times 1) = \pi$ $\frac{d^2V}{dr^2} = 6\pi - 12\pi r$ When $r = 1$, $\frac{d^2V}{dr^2} = -6\pi < 0$ Therefore $V = \pi$ is maximum volume.  	Correct expression for $\frac{dV}{dr}$.	Correct expression for $\frac{dV}{dr}$ and finds $r = 1$.	**T1**: Correct expression for $\frac{dV}{dr}$ and shows that $V = \pi$ but does not prove it is the maximum volume with either the first or second derivative test. **T2**: Correct expression for $\frac{dV}{dr}$ and correct proof.

NØ	**N1**	**N2**	**A3**	**A4**	**M5**	**M6**	**E7**	**E8**
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	T1	T2 or two T1

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
TWO (a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 5(1-x^2)^4 \times (-2x)$	Correct derivative.		
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(x+1)2x - x^2}{(x+1)^2}$ $= \frac{x^2+2x}{(x+1)^2}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow x(x+2) = 0$ $x = 0$ or $x = -2$	Correct solutions with correct derivative.		
(c)	$y = (x^2+3x+2)\cos 3x$ $\frac{\mathrm{d}y}{\mathrm{d}x} = (2x+3)\cos 3x - (x^2+3x+2)3\sin 3x$ Crosses y-axis $\Rightarrow x = 0,\ y = 2,\ \frac{\mathrm{d}y}{\mathrm{d}x} = 3$ Normal gradient is $\frac{-1}{3}$ Equation of normal: $y - 2 = \frac{-1}{3}(x-0)$ $y = \frac{-1}{3}x + 2$ $3y + x - 6 = 0$	Correct derivative.	Correct solution with correct derivative.	
(d)	$\frac{\mathrm{d}V}{\mathrm{d}t} = 60$ $V = \frac{4}{3}\pi r^3$ $\frac{\mathrm{d}V}{\mathrm{d}r} = 4\pi r^2$ $\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\mathrm{dV}}{\mathrm{d}t} \times \frac{\mathrm{d}r}{\mathrm{d}V}$ $= \frac{60}{4\pi r^2}$ $= \frac{15}{\pi r^2}$ $r = 15 \Rightarrow \frac{\mathrm{d}r}{\mathrm{d}t} = \frac{15}{\pi 15^2}$ $= \frac{1}{15\pi}\ (= 0.0212)\ \text{cm s}^{-1}$	Correct expression for $\frac{\mathrm{d}r}{\mathrm{d}t}$.	Correct solution with correct $\frac{\mathrm{d}r}{\mathrm{d}t}$.	

(e)	$y=\sqrt{2x-4}$ $\frac{dy}{dx}=\frac{1}{\sqrt{2x-4}}$ Gradient of tangent $=\frac{y-1}{x+2}$ $\frac{1}{\sqrt{2x-4}}=\frac{\sqrt{2x-4}-1}{x+2}$ $x+2=2x-4-\sqrt{2x-4}$ $\sqrt{2x-4}=x-6$ $2x-4=x^2-12x+36$ $x^2-14x+40=0$ $(x-4)(x-10)=0$ $x=4$ or $x=10$ Rejecting $x=4$ by checking the surd equation $x=10$ $\sqrt{16}=4$ True $x=4$ $\sqrt{4}=-2$ False One solution: $x=10$ Therefore, the coordinates of point P are $(10,4)$ OR Rejecting $x=4$ by checking the gradient: At $(10,4)$, $\frac{dy}{dx}=\frac{1}{\sqrt{16}}=\frac{1}{4}$ Gradient: $\frac{y-1}{x+2}=\frac{3}{12}=\frac{1}{4}$ At $(4,2)$, $\frac{dy}{dx}=\frac{1}{\sqrt{4}}=\frac{1}{2}$ Gradient: $\frac{y-1}{x+2}=\frac{1}{6}$ One solution: $x=10$ Therefore, the coordinates of point P are $(10,4)$	Correct derivative: $\frac{dy}{dx}=\frac{1}{\sqrt{2x-4}}$	Correct derivative: $\frac{dy}{dx}=\frac{1}{\sqrt{2x-4}}$ and $\sqrt{2x-4}=x-6$	**T1:** Correct solution with correct derivative: P (10,4) without any justification for $x\neq 4$ **T2:** Correct solution with correct derivative: P (10,4) $x\neq 4$ must be justified with respect to either the surd equation or the gradient of the tangent.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	T1	T2

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
THREE (a)	$\frac{dy}{dx}=\frac{(x^2+1)(-\text{cosec}^2 x)-(\cot x)(2x)}{(x^2+1)^2}$	Correct derivative.		
(b)	$\frac{dy}{dx}=\frac{2}{\sqrt{x}}-1$ At stationary point, derivative = 0. $\frac{2}{\sqrt{x}}=1$ $x=4$ Coordinates are $(4,6)$.	Correct solution with correct derivative.		
(c)	$\frac{dy}{dx}=\frac{(x^2+4)-x(2x)}{(x^2+4)^2}$ $=\frac{4-x^2}{(x^2+4)^2}$ Increasing when $\frac{dy}{dx}>0$ $\frac{4-x^2}{(x^2+4)^2}>0$ $4-x^2>0$ $-2<x<2$	Correct $\frac{dy}{dx}$	Correct $\frac{dy}{dx}$ and identifies –2 and 2 as the boundaries of the interval required.	**T1:** Correct solution with correct derivative.
(d)	$\frac{dy}{dx}=\frac{(4x-k)4-(4x+k)4}{(4x-k)^2}$ $=\frac{16x-4k-16x-4k}{(4x-k)^2}$ $=\frac{-8k}{(4x-k)^2}$ When $x=3, \frac{dy}{dx}=\frac{-8}{27}$ $\frac{-8k}{(12-k)^2}=\frac{-8}{27}$ $\frac{k}{(12-k)^2}=\frac{1}{27}$ $27k=144-24k+k^2$ $k^2-51k+144=0$ $k=48$ or $k=3$	Correct derivative.	Correct solution with correct derivative.	

(e)	$\cos\theta = \frac{h}{S}$ $S^2 = h^2 + r^2$ $S = \sqrt{h^2 + r^2}$ k and r are constant $I = \frac{k\cos\theta}{S^2}$ $I = \frac{k\frac{h}{S}}{S^2}$ $= \frac{kh}{S^3}$ $I = \frac{kh}{\left(h^2 + r^2\right)^{\frac{3}{2}}}$ $\frac{\mathrm{d}I}{\mathrm{d}h} = \frac{\left(h^2 + r^2\right)^{\frac{3}{2}}k - kh\left(\frac{3}{2}\right)\left(h^2 + r^2\right)^{\frac{1}{2}}(2h)}{\left(h^2 + r^2\right)^3}$ $\frac{\mathrm{d}I}{\mathrm{d}h} = \frac{k\left(h^2 + r^2\right)^{\frac{3}{2}} - 3kh^2\left(h^2 + r^2\right)^{\frac{1}{2}}}{\left(h^2 + r^2\right)^3}$ $\frac{\mathrm{d}I}{\mathrm{d}h} = \frac{k\left(h^2 + r^2\right)^{\frac{1}{2}}\left(h^2 + r^2 - 3h^2\right)}{\left(h^2 + r^2\right)^3}$ $\frac{\mathrm{d}I}{\mathrm{d}h} = \frac{k\left(r^2 - 2h^2\right)}{\left(h^2 + r^2\right)^{\frac{5}{2}}}$ $\frac{\mathrm{d}I}{\mathrm{d}h} = 0 \Rightarrow k\left(r^2 - 2h^2\right) = 0$ $2h^2 = r^2$ $h^2 = \frac{r^2}{2}$ $h = \frac{r}{\sqrt{2}}$		Correct expression for $\frac{\mathrm{d}I}{\mathrm{d}h}$	T2: Correct proof with correct derivative

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	T1	T2

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 6	7– 12	13 – 18	19 – 24