Assessment Schedule – 2022

Calculus: Apply differentiation methods in solving problems (91578)

Evidence Statement

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
ONE (a)	$\frac{dy}{dx} = 2\ln x \cdot \frac{1}{x}$	Correct derivative		
(b)	$f(x) = \frac{x^2+1}{x}$ $f'(x) = \frac{x.(2x)-(x^2+1)}{x^2}$ $\frac{x.(2x)-(x^2+1)}{x^2} = 0$ $2x^2 - x^2 - 1 = 0$ $x^2 = 1$ $x = \pm 1$ OR $f(x) = x + x^{-1}$ $f'(x) = 1 - x^{-2}$ $1 - \frac{1}{x^2} = 0$ $x^2 - 1 = 0$ $x = \pm 1$	Correct solution with correct derivative Must have both solutions: $x = \pm 1$		
(c)	$y = \sqrt{x+2}$ $\frac{dy}{dx} = \frac{1}{2}(x+2)^{\frac{-1}{2}}$ $= \frac{1}{2\sqrt{x+2}}$ At $x = 0, \frac{dy}{dx} = \frac{1}{2\sqrt{2}}$ and $y = \sqrt{2}$ Equation of normal is $y = -2\sqrt{2}x + \sqrt{2}$ x intercept $(y = 0)$ $\quad 0 = -2\sqrt{2}x + \sqrt{2}$ $x = \frac{1}{2}$ Coordinate of P *is* $\left(\frac{1}{2}, 0\right)$	Correct derivative with $\frac{dy}{dx}$ evaluated at $x = 0$.	Correct solution with correct $\frac{dy}{dx}$. Accept $x = \frac{1}{2}$. $y = 0$ can be implied in the working.	

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
(d)	$\frac{dx}{dt}=3$ $\frac{dy}{dt}=3-\frac{3}{3t-1}$ $=\frac{3(3t-1)-3}{3t-1}$ $=\frac{9t-6}{3t-1}$ $\frac{dy}{dx}=\frac{9t-6}{3t-1}\times\frac{1}{3}$ $=\frac{3t-2}{3t-1}$ $\frac{dy}{dx}=\frac{1}{2}$ $\frac{3t-2}{3t-1}=\frac{1}{2}$ $6t-4=3t-1$ $3t=3$ $t=1$ $x=5 \quad y=3-\ln 2$ or 2.307	Correct expression for $\frac{dy}{dx}$.	Correct solution with correct $\frac{dy}{dx}$.	

	Expected coverage	**Achievement (u)**	**Merit (r)**	**Excellence (t)**
(e)	$y = \mathrm{e}^{px^2}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = 2px\mathrm{e}^{px^2}$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2px.2px.\mathrm{e}^{px^2} + 2p.\mathrm{e}^{px^2}$ $= 2p\mathrm{e}^{px^2}(2px^2+1)$ At a point of inflection, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0$ $2p\mathrm{e}^{px^2}(2px^2+1) = 0$ **Equation 1** $2p\mathrm{e}^{px^2} = 0$ $p\mathrm{e}^{px^2} = 0$ $px^2 = \ln 0$ No solution as ln 0 is defined. OR $2p\mathrm{e}^{px^2} = 0$ has no solutions because $2p\mathrm{e}^{px^2} > 0$ for all values of x since $\mathrm{e}^{px^2} > 0$ for all values of a and p is a positive **Equation 2** $2px^2 + 1 = 0$ $x^2 = \frac{-1}{2p}$ $x = \sqrt{\frac{-1}{2p}}$ $2px^2 + 1 = 0$ has no real solutions because p is a positive real constant, $\frac{-1}{2p}$ is negative and there is not a real solution when you take the square root of a negative number. OR $2px^2 + 1 = 0$ has no real solutions because $2px^2 + 1 = 0$ is always greater than zero because p is a positive real constant and x^2 is always greater than or equal to zero OR $2px^2 + 1 = 0$ has no real solutions because the discriminant is less than zero. $b^2 - 4ac = 0 - 4(2p)(1) = -8p$ Since p is a positive real constant. Therefore, there are no solutions to $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0$ nd and $y = \mathrm{e}^{px^2}$ has no points of inflection.	Correct $\frac{\mathrm{d}y}{\mathrm{d}x}$.	Correct $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$.	**T1** Correct $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ with one part of the equation set to zero and the reason for there being no real solutions given for EITHER $2p\mathrm{e}^{px^2} = 0$ OR $2px^2 + 1 = 0$ **T2** Correct proof with correct derivatives Both parts of the equation set to zero and the reason for there being no real solutions given for both equations

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	T1	T2

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
TWO (a)	$f'(x) = 4(5x-3)\cos 4x + 5\sin 4x$	Correct derivative.		
(b)	$y = \left(3x^2-2\right)^3$ $\frac{\mathrm{d}y}{\mathrm{d}x} = 3\left(3x^2-2\right)^2\left(6x\right)$ At $x=2, \frac{\mathrm{d}y}{\mathrm{d}x} = 3600$	Correct solution with correct derivative.		
(c)	$d(t) = \frac{t^2-6}{2t^3}$ $v(t) = \frac{2t^3\left(2t\right)-(t^2-6)\left(6t^2\right)}{4t^6}$ $v(t) = \frac{36t^2-2t^4}{4t^6}$ $v(t) = \frac{18-t^2}{2t^4}$ Stationary point when $v(t)=0$ $18-t^2=0$ $t=\sqrt{18}\ \ (=4.24)$	Correct derivative.	Correct solution with correct derivative.	
(d)	$y = 6\mathrm{e}^{1-0.5x}$ Area $= 6x\mathrm{e}^{1-0.5x}$ $A'(x) = 6\mathrm{e}^{1-0.5x} + 6x\mathrm{e}^{1-0.5x} \times -0.5$ $= 6\mathrm{e}^{1-0.5x} - 3x\mathrm{e}^{1-0.5x}$ $= 3\mathrm{e}^{1-0.5x}(2-x)$ At maximum, $A'(x)=0$ $x=2$ Area $= 12\mathrm{e}^{1-1} = 12$	Correct derivative of $A(x)$.	Correct solution with correct derivative.	

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
(e)	$(y-5)^2=16(x-2)$ Method A $y-5=4\sqrt{x-2}$ $y=4\sqrt{x-2}+5$ $\frac{dy}{dx}=\frac{2}{\sqrt{x-2}}$ $\frac{dy}{dx}=1$ $\frac{2}{\sqrt{x-2}}=1$ $\sqrt{x-2}=2$ $x-2=4$ $x=6$ $y=13$ Method B $2(y-5)\frac{dy}{dx}=16$ $\frac{dy}{dx}=\frac{8}{y-5}$ $\frac{dy}{dx}=1$ $\frac{8}{y-5}=1$ $y=13$ $64=16x-32$ $x=6$ Equation of tangent $y-13=1(x-6)$ $y=x+7$ Axis intercepts: (0,7) and (-7, 0) Distance RS = $\sqrt{7^2+7^2}$ $=\sqrt{49\times 2}$ $=7\sqrt{2}$	Correct $\frac{dy}{dx}$.	Correct x and y values found (6,13) with correct $\frac{dy}{dx}$.	**T1** Finds equation of tangent and both axis intercepts with correct $\frac{dy}{dx}$. **T2** Correct solution with correct $\frac{dy}{dx}$.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	T1	T2

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
THREE (a)	$\frac{dy}{dx} = e^{4\sqrt{x}}.2x^{\frac{-1}{2}}$	Correct derivative		
(b)(i) (ii) (iii)	$x = -4, -1, 1$ $x = -3, x > 1$ 3	Two correct parts of question Three (b)		
(c)	$V = \pi\left(\frac{3}{2}h^2 + 3h\right)$ $\frac{dV}{dh} = \pi(3h+3)$ $\frac{dV}{dt} = 20$ $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$ $= \frac{1}{\pi(3h+3)} \times 20$ At $h = 3, \frac{dh}{dt} = \frac{20}{12\pi}$ $= \frac{5}{3\pi} = 0.531\,\text{cm s}^{-1}$	Correct expressions for $\frac{dV}{dh}$ and $\frac{dV}{dt}$. $\frac{dV}{dt}$. can be implied by the expression for $\frac{dh}{dt}$.	Correct solution with correct derivative for $\frac{dh}{dt}$.	

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
(d)	$y = 9x - 2 + \dfrac{3}{3x-1}$ $\dfrac{dy}{dx} = 9 - 3(3x-1)^{-2} \times 3$ $= 9 - \dfrac{9}{(3x-1)^2}$ Stationary point $\dfrac{dy}{dx} = 0$ $9 - \dfrac{9}{(3x-1)^2} = 0$ $9 = \dfrac{9}{(3x-1)^2}$ $(3x-1)^2 = 1$ $3x - 1 = \pm 1$ $x = \dfrac{1 \pm 1}{3}$ $x = 0$ or $x = \dfrac{2}{3}$ $\dfrac{d^2y}{dx^2} = \dfrac{54}{(3x-1)^3}$ $x = 0 \quad \dfrac{d^2y}{dx^2} = \dfrac{54}{(-1)^3} < 0$ Local max at $x = 0$ $x = \dfrac{2}{3} \quad \dfrac{d^2y}{dx^2} = \dfrac{54}{(1)^3} > 0$ Local min at $x = \dfrac{2}{3}$	Correct derivative.	Correct solution with correct derivative. The nature of each turning point stated but not determined using a calculus method.	**T1** Correct solution with correct derivative. The nature of each turning point determined with a first or second derivative test.

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
(e)	Total time = time (HP) + time (PS) **Method A** Let x = distance PQ $T=\frac{4-x}{10}+\frac{\sqrt{x^2+4}}{6}$ $\frac{dT}{dx}=\frac{-1}{10}+\frac{\frac{1}{2}(x^2+4)^{\frac{-1}{2}}.2x}{6}$ $\frac{dT}{dx}=\frac{-1}{10}+\frac{x}{6\sqrt{x^2+4}}$ For maximum/minimum time, $\frac{dT}{dx}=0$ $\frac{1}{10}=\frac{x}{6\sqrt{x^2+4}}$ $6\sqrt{x^2+4}=10x$ $\sqrt{x^2+4}=\frac{10}{6}x$ $x^2+4=\frac{25}{9}x^2$ $4=\frac{16}{9}x^2$ $\frac{36}{16}=x^2$ $x=1.5$ $4-1.5=2.5$ Megan should travel 2.5 km along the path before cutting across the park. **Method B** Let x = distance HP $T=\frac{x}{10}+\frac{\sqrt{\left((4-x)^2+4\right)}}{6}$ $\frac{dT}{dx}=\frac{1}{10}+\frac{(x-4)}{6\sqrt{x^2-8x+20}}$ $\frac{dT}{dx}=0$ $\frac{1}{10}+\frac{(x-4)}{6\sqrt{x^2-8x+20}}=0$ $5(x-4)=-3\sqrt{x^2-8x+20}$ $25(x^2-8x+16)=9(x^2-8x+20)$ $25x^2-200x+400=9x^2-72x+180$ $16x^2-128x+220=0$ $x=2.5$ or 5.5 Since $x<4$, $x=2.5$ km		Correct $\frac{dT}{dx}$.	**T1** Method A x = 1.5 found with correct derivative OR **T1** Method B x = 2.5 or 5.5 found (5.5 not discarded) with correct derivative. **T2** Correct solution with correct derivative.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	T1	T2 or two of T1

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 7	8 – 13	14 – 19	20 – 24