Assessment Schedule – 2023

Calculus: Apply differentiation methods in solving problems (91578)

Evidence Statement

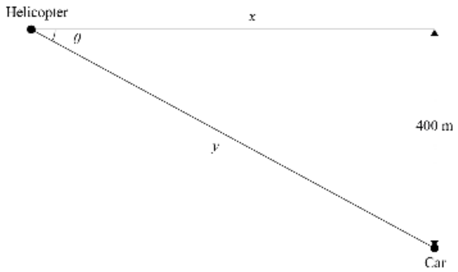
	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
ONE (a)	$\frac{dy}{dx} = \frac{1}{2}(3x-2)^{-\frac{1}{2}} \times 3 = \frac{3}{2}(3x-2)^{-\frac{1}{2}}$	• Correct derivative.		
(b)	$f'(t) = t^2\left(2e^{2t}\right) + e^{2t}\left(2t\right)$ $= 2t^2e^{2t} + 2te^{2t}$ $= 2te^{2t}\left(t+1\right)$ $f'(1.5) = 3e^3\left(2.5\right)$ $= 7.5e^3 = 150.64$	• Correct derivative. AND Correct rate of change.		
(c)	$\frac{dy}{dx} = -6(x+1)^{-4}$ $= \frac{-6}{\left(x+1\right)^4}$ When $x = 1, \frac{dy}{dx} = -\frac{3}{8}$ or -0.375 $\frac{-6}{\left(x+1\right)^4} = -\frac{3}{8}$ $3\left(x+1\right)^4 = 48$ $\left(x+1\right)^4 = 16$ $x+1 = \pm 2$ $x = 1$ or -3 $\therefore$ Second tangent touches the curve when $x = -3$	• Correct derivative for $\frac{dy}{dx}$. AND Correct gradient of $\frac{3}{8}$ found.	• Finds the correct value of x for the second tangent, with evidence of derivative.	

(d)	$x = 4\cos\theta$ and $y = 4\sin\theta$ $\frac{\mathrm{d}x}{\mathrm{d}\theta} = -4\sin\theta$ and $\frac{\mathrm{d}y}{\mathrm{d}\theta} = 4\cos\theta$ $\frac{dy}{dx} = \frac{4\cos\theta}{-4\sin\theta} = -\frac{x}{y} = -\frac{p}{q}$ OR Equation of circle: $x^2 + y^2 = 16$ Differentiating implicitly gives $2x + 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{x}{y}$ At (p,q), the gradient is $-\frac{p}{q}$ Equation of tangent: $y - q = -\frac{p}{q}(x - p)$ $qy - q^2 = -px + p^2$ $px + qy = p^2 + q^2$ as required.	• Correct derivative for $\frac{\mathrm{d}y}{\mathrm{d}x}$. • $\frac{\mathrm{d}y}{\mathrm{d}x}$ can be expressed in terms of θ or x,y or p,q).	• Proof completed, with correct .	

(e)	Area of triangle $= \frac{1}{2}xy$ $A = \frac{1}{2}x\left(x(x-2m)^2\right)$ $= \frac{1}{2}x^2(x-2m)^2$ $\frac{\mathrm{d}A}{\mathrm{d}x} = \frac{1}{2}x^2(2(x-2m)) + (x-2m)^2$ $= x^2(x-2m) + x(x-2m)^2$ $= x(x-2m)\left(x+(x-2m)\right)$ $= x(x-2m)(2x-2m)$ $= 2x(x-2m)(x-m)$ OR $A = \frac{1}{2}x^2(x-2m)^2$ $= \frac{1}{2}x^4 - 2mx^3 + 2m^2x^2$ $\frac{\mathrm{d}A}{\mathrm{d}x} = 2x^3 - 6mx^2 + 4m^2x$ $= 2x\left(x^2 - 3mx + 2m^2\right)$ $= 2x(x-2m)(x-m)$ $\frac{\mathrm{d}A}{\mathrm{d}x} = 0 \Rightarrow 2x(x-2m)(x-m) = 0$ $x = 0$ or $x = 2m$ or $x = m$ Since $0 < x < 2m$ the area is a maximum when $x = m$ Maximum area of triangle: $A(m) = \frac{1}{2}m^2\left(m-2m\right)^2$ $= \frac{1}{2}m^4$ This is $\frac{3}{8}$ of the total shaded area since $\frac{3}{8} \times \frac{4m^3}{3} = \frac{1}{2}m^4$	• Correct derivative.	• Correct derivative. AND $x = m$ found.	T1 Maximum area, $A = \frac{1}{2}m^4$ found with correct $\frac{\mathrm{d}A}{\mathrm{d}x}$. OR Correct solution but with one minor error. T2 Correct solution with correct $\frac{\mathrm{d}A}{\mathrm{d}x}$ showing the calculation of the correct proportion of total shaded area..

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t	2t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
TWO (a)	$f'(x) = \frac{(\cos x)(2x)-\left(x^2\right)(-\sin x)}{\cos^2 x}$ $= \frac{2x\cos x + x^2 \sin x}{\cos^2 x}$ $= \frac{x(2\cos x + x\sin x)}{\cos^2 x}$	• Correct derivative.		
(b)	$\frac{dy}{dx} = -2\operatorname{cosec}^2(2x)$ When $x = \frac{\pi}{12}$ $\frac{dy}{dx} = \frac{-2}{\sin^2\left(\frac{\pi}{6}\right)}$ $= -8$	• Correct derivative. AND Correct gradient of –8 found.		
(c)	$f'(x) = \frac{(x^2+2x)e^x - e^x(2x+2)}{(x^2+2x)^2}$ $= \frac{e^x\left((x^2+2x)-(2x+2)\right)}{(x^2+2x)^2}$ $= \frac{e^x\left(x^2-2\right)}{(x^2+2x)^2}$ $f'(x) = 0 \Rightarrow e^x\left(x^2-2\right) = 0$ $e^x \neq 0$ $x^2 - 2 = 0$ $x = \pm\sqrt{2}$ or $x = \pm 1.41$	• Correct derivative.	• Correct both values of x found, with evidence of derivative.	
(d)	$f'(x) = 3x^2 \cdot \frac{1}{x} + \ln x \cdot (6x)$ $= 3x + 6x\ln x$ $f''(x) = 3 + 6x \cdot \frac{1}{x} + \ln x(6)$ $= 9 + 6\ln x$ $f''(x) = 0 \Rightarrow 9 + 6\ln x = 0$ $\ln x = -1.5$ $x = e^{-1.5}$ or $x = 0.223$	• Correct $f'(x)$.	• Correct $f'(x)$. AND Correct $f''(x)$. AND Correct x-value.	

(e)	 Let x = horizontal distance between the helicopter and the car. Let y = direct distance between the helicopter and the car. Given: $\frac{d\theta}{dt} = 0.002\ rad\ s-1$ $\tan\theta = \frac{400}{x}$ $x = 400\cot\theta$ $\frac{dx}{d\theta} = -400\,\text{cosec}^2\theta$ $= \frac{-400}{\sin^2\theta}$ $\frac{dx}{dt} = \frac{dx}{d\theta} \times \frac{d\theta}{dt}$ $= \frac{-400}{\sin^2\theta} \times 0.002$ $= \frac{-0.8}{\sin^2\theta}$ When $y = 2500, \sin\theta = \frac{400}{2500}$ $\theta = 0.1607$ rad $\frac{dx}{dt} = \frac{-0.8}{\sin^2(0.1607}$ $= 31.25$ When the helicopter is travelling at 72 m s^{-1}, The speed of the car $= 72 - 31.25$ $= 40.75$ m s^{-1} (= 146.7 km/hr)	• Finds $\frac{dx}{d\theta}$. .	• Finds an expression for $\frac{dx}{dt}$.	T1 Finds the value for $\frac{dx}{dt} = -31.25$ With correct derivatives. OR Finds correct solution but with one minor error. T2 Finds $\frac{dx}{dt} = -31.25$ with correct derivatives. AND The speed of the car = 40.76 m s^{-1}.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t	2t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
THREE (a)	$\dfrac{dy}{dx} = \dfrac{2x - 4x^3}{x^2 - x^4 + 1}$	• Correct derivative.		
(b)(i) (ii) (iii)	$x = 8$ $x = -4$ The limit does not exist.	• 2 out of 3 correct responses.		
(c)	$\dfrac{dx}{dt} = \sqrt{2}\pi\cos\left(\dfrac{\pi t}{5}\right)$ $\dfrac{dy}{dt} = \sqrt{2}\pi\sin\left(\dfrac{\pi t}{5}\right) \Rightarrow$ $\dfrac{dy}{dx} = \tan\left(\dfrac{\pi t}{5}\right)$ When $t = 6.25 \Rightarrow$ $\dfrac{dy}{dx} = \tan(1.25\pi) = 1$ Normal gradient: $m = -1$	• Correct expression for $\dfrac{dx}{dt}$ AND $\dfrac{dy}{dt}$.	• Gradient of the normal found.	
(d)	$f(x) = x^{-1} - 2x^{-3}$ $f'(x) = -x^{-2} + 6x^{-4} = 0$ $\dfrac{6}{x^4} = \dfrac{1}{x^2}$ $6x^2 - x^4 = 0$ $x^2(6 - x^2) = 0$ $x \neq 0$ *so* $x = \pm\sqrt{6}$ $f''(x) = 2x^{-3} - 24x^{-5}$ $f''(\sqrt{6}) = -0.136 < 0$ i.e. maximum $f''(-\sqrt{6}) = 0.136 > 0$ i.e. minimum Maximum at $\left(\sqrt{6}, \dfrac{\sqrt{6}}{9}\right) = (2.45, 0.2722)$ Minimum at $\left(-\sqrt{6}, \dfrac{-\sqrt{6}}{9}\right) = (-2.45, -0.2722)$	• Correct values of x found, with evidence of derivative.	• Co-ordinates and nature of the two turning points found and distinguished, with evidence of a calculus method.	

(d)	$f(x) = x^{-1} - 2x^{-3}$ $f'(x) = -x^{-2} + 6x^{-4}$ $= \frac{-1}{x^2} + \frac{6}{x^4}$ $f'(x) = 0 \Rightarrow \frac{-1}{x^2} + \frac{6}{x^4} = 0$ $x^4 - 6x^2) = 0$ $x^2(x^2 - 6) = 0$ $x = 0$ not possible $x^2 - 6 = 0$ $x = \pm\sqrt{6}$ or $x = \pm 2.45$ **Second derivative test :** $f''(x) = 2x^{-3} - 24x^{-5}$ $= \frac{2}{x^3} - \frac{24}{x^5}$ $f''(\sqrt{6}) = -0.136 = -\frac{\sqrt{6}}{18}$ Since $f''(\sqrt{6}) < 0, x = \sqrt{6}$ is a local maximum. $f''(\sqrt{6}) = -0.136 = -\frac{\sqrt{6}}{18}$ Since $f''(\sqrt{6}) > 0, x = -\sqrt{6}$ is a local minimum. Maximum turning point when $x = \sqrt{6}$. Minimum turning point when $x = -\sqrt{6}$.	• Correct derivative. AND Correct two values of x found (not x = 0).	• x-coordinates of both stationary points found. AND The nature of the two turning points found with a correct first or second derivative test. Not required: $f\left(\sqrt{6}\right) = \frac{\sqrt{6}}{9}$ $= 0.272$ $f\left(-\sqrt{6}\right) = \frac{\sqrt{6}}{9}$ $= -0.272$	

(e)	$y = \frac{a}{2}\left(e^{\frac{x}{a}} + e^{\frac{-x}{a}}\right)$ $= \left(\frac{a}{2}e^{\frac{x}{a}} + \frac{a}{2}e^{\frac{-x}{a}}\right)$ $\frac{dy}{dx} = \frac{1}{2}e^{\frac{x}{a}} - \frac{1}{2}e^{\frac{-x}{a}}$ $\frac{d^2y}{dx^2} = \frac{1}{2a}e^{\frac{x}{a}} + \frac{1}{2a}e^{\frac{-x}{a}}$ $\left(\frac{dy}{dx}\right)^2 = \frac{1}{4}e^{\frac{2x}{a}} + \frac{1}{4}e^{\frac{-2x}{a}} - \frac{1}{2}$ #(1) $\text{LHS} = a\frac{d^2y}{dx^2}$ $= \frac{1}{2}e^{\frac{x}{a}} + \frac{1}{2}e^{\frac{-x}{a}}$ #(2) $\text{RHS} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ $= \sqrt{1 + \frac{1}{4}e^{\frac{2x}{a}} + \frac{1}{4}e^{\frac{-2x}{a}} - \frac{1}{2}}$ $= \sqrt{\frac{1}{4}e^{\frac{2x}{a}} + \frac{1}{4}e^{\frac{-2x}{a}} + \frac{1}{2}}$ #(3) $= \sqrt{\left(\frac{1}{4}e^{\frac{2x}{a}} + e^{\frac{-2x}{a}}\right)^2}$ $= \frac{1}{2}\left(e^{\frac{x}{a}} - e^{\frac{-x}{a}}\right)$ $= a\frac{d^2y}{dx^2}$ $=$ LHS as required	• Correct expression for $\frac{dy}{dx}$.	• Correct expression for $\frac{d^2y}{dx^2}$. AND Evidence of progress with substitution into the differential equation Reaches either stage #(1) OR stage #(2).	T1 • Reaches both stage #(2) AND stage #(3) with correct derivatives. OR Correct solution but with one minor error. T2 • Correct proof with correct derivatives.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t	2t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 7	8– 12	13 – 18	19 – 24