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SUPERVISOR'S USE ONLY

Level 3 Calculus, 2017

91578 Apply differentiation methods in solving problems

9.30 a.m. Thursday 23 November 2017
 Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply differentiation methods in solving problems.	Apply differentiation methods, using relational thinking, in solving problems.	Apply differentiation methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

ASSESSOR'S USE ONLY

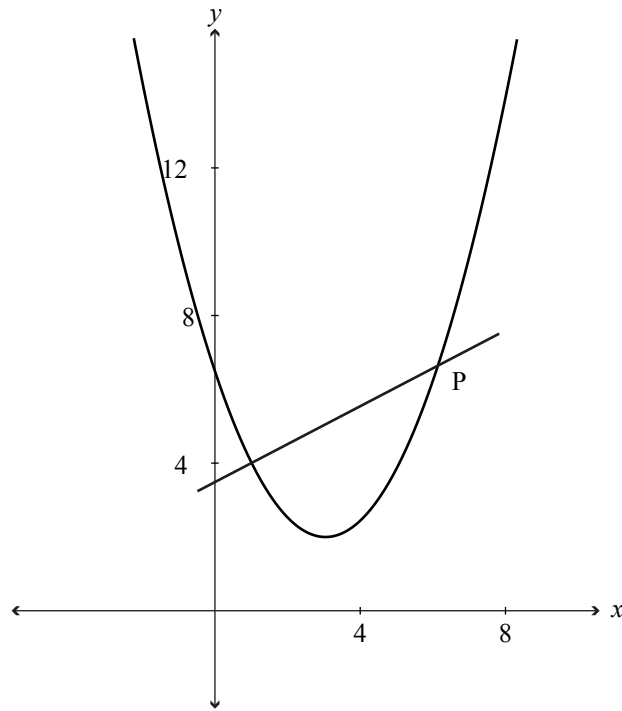
QUESTION ONEASSESSOR'S
USE ONLY

- (a) Differentiate $y = \sqrt{x} + \tan(2x)$.

- (b) Find the gradient of the tangent to the curve $y = \frac{e^{2x}}{x+2}$ at the point where $x = 0$.

You must use calculus and show any derivatives that you need to find when solving this problem.

- (c) The normal to the parabola $y = 0.5(x - 3)^2 + 2$ at the point $(1, 4)$ intersects the parabola again at the point P.



Find the x -coordinate of point P.

You must use calculus and show any derivatives that you need to find when solving this problem.

- (d) A curve is defined parametrically by the equations $x = \sqrt{t+1}$ and $y = \sin 2t$.

Find the gradient of the tangent to the curve at the point when $t = 0$.

You must use calculus and show any derivatives that you need to find when solving this problem.

- You must use calculus and show any derivatives that you need to find when solving this problem.*

QUESTION TWOASSESSOR'S
USE ONLY

- (a) Differentiate $y = 2(x^2 - 4x)^5$.

You do not need to simplify your answer.

- (b) The percentage of seeds germinating depends on the amount of water applied to the seedbed that the seeds are sown in, and may be modelled by the function:

$$P(w) = 96 \ln(w + 1.25) - 16w - 12$$

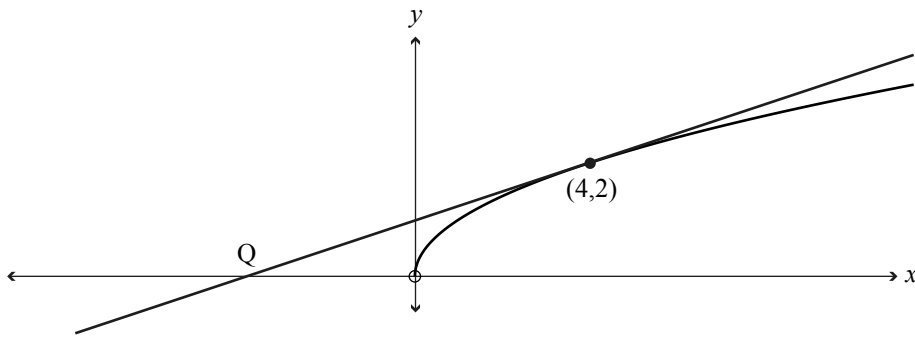
where P is the percentage of seeds that germinate and

w is the daily amount of water applied (litres per square metre of seedbed), with $0 \leq w \leq 15$.

Find the amount of water that should be applied daily to maximise the percentage of seeds germinating.

You must use calculus and show any derivatives that you need to find when solving this problem.

- (c) The tangent to the curve $y = \sqrt{x}$ is drawn at the point $(4,2)$.



Find the co-ordinates of the point Q where the tangent intersects the x -axis.

You must use calculus and show any derivatives that you need to find when solving this problem.

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- A Cartesian coordinate system with a horizontal x-axis and a vertical y-axis. The x-axis is labeled 'x' at its right end, and the y-axis is labeled 'y' at its top end. The origin is marked with a small circle. The x-axis has tick marks and labels at 2, 4, 6, 8, and 10. The y-axis has tick marks and labels at 2 and 4. A smooth, increasing curve starts at the origin and extends into the first quadrant. A point on the curve is marked with a solid black dot and labeled 'P (x,y)'.

You must use calculus and show any derivatives that you need to find when solving this problem.

-
- A diagram showing a semicircle with a horizontal diameter and a vertical radius line. A rectangle is inscribed within the semicircle, with its base on the diameter. The left side of the rectangle is labeled $-x$ and the right side is labeled x . The top corners of the rectangle lie on the arc of the semicircle. The origin of the coordinate system is marked with a small circle at the center of the diameter.

You must use calculus and show any derivatives that you need to find when solving this problem.

QUESTION THREEASSESSOR'S
USE ONLY

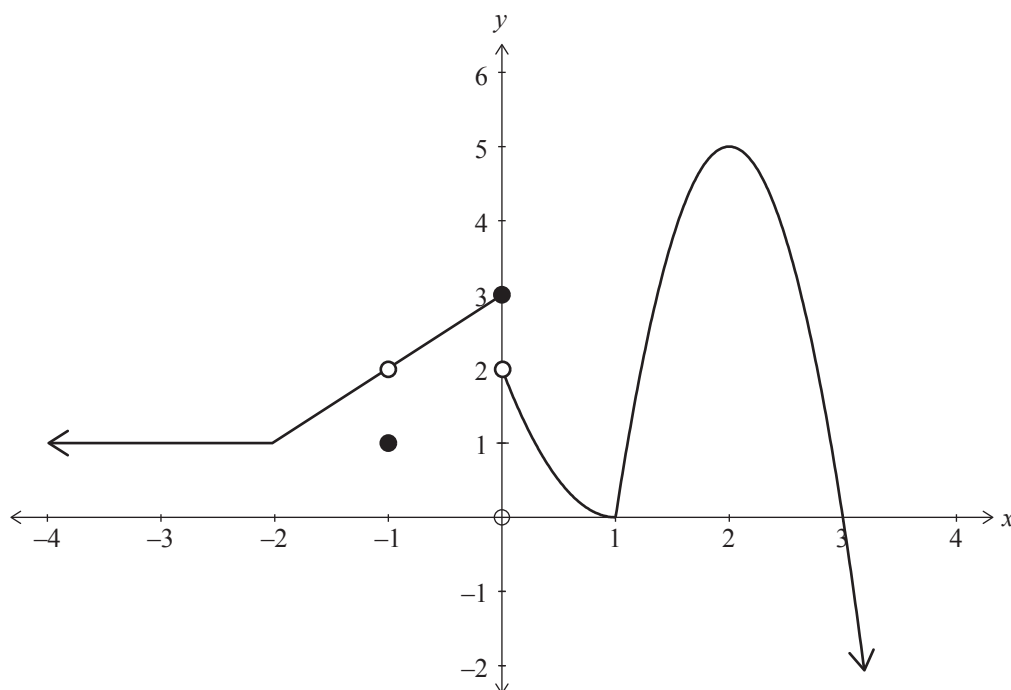
- (a) Differentiate $y = x \ln(3x - 1)$.

You do not need to simplify your answer.

- (b) Find the gradient of the curve $y = \frac{1}{x} - \frac{1}{x^2}$ at the point $\left(2, \frac{1}{4}\right)$.

You must use calculus and show any derivatives that you need to find when solving this problem.

- (c) The graph below shows the function $y = f(x)$.



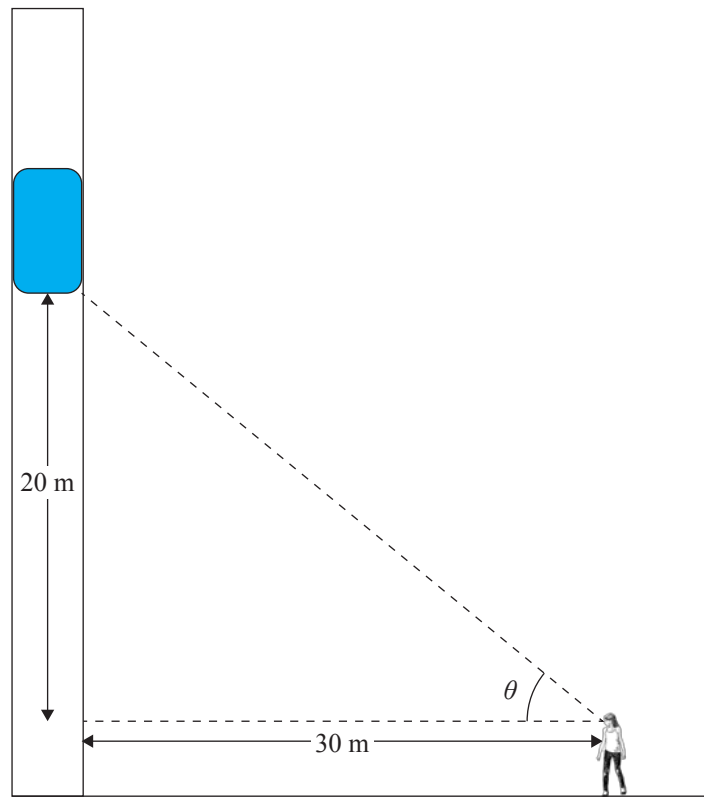
For the function above:

- (i) Find the value(s) of x that meet the following conditions:

- (1) $f'(x) = 0$: _____
- (2) $f(x)$ is continuous but not differentiable: _____
- (3) $f(x)$ is not continuous: _____
- (4) $f''(x) < 0$: _____

- (ii) What is the value of $\lim_{x \rightarrow -1} f(x)$? _____

State clearly if the value does not exist.



You must use calculus and show any derivatives that you need to find when solving this problem.

(e) For the function $y = e^x \cos kx$:

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

(ii) Find all the value(s) of k such that the function $y = e^x \cos kx$ satisfies the equation

$$\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + 2y = 0 \text{ for all values of } x.$$

Extra paper if required.
Write the question number(s) if applicable.

QUESTION
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