SUPERVISOR'S USE ONLY

3

91578

915780

Draw a cross through the box (☒) if you have NOT written in this booklet

+

NZQA | Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Level 3 Calculus 2023

91578 Apply differentiation methods in solving problems

Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply differentiation methods in solving problems.	Apply differentiation methods, using relational thinking, in solving problems.	Apply differentiation methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (DO NOT WRITE IN THIS AREA DO NOT WRITE). This area will be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

© New Zealand Qualifications Authority, 2023. All rights reserved.
No part of this publication may be reproduced by any means without the prior permission of the New Zealand Qualifications Authority.

QUESTION ONE

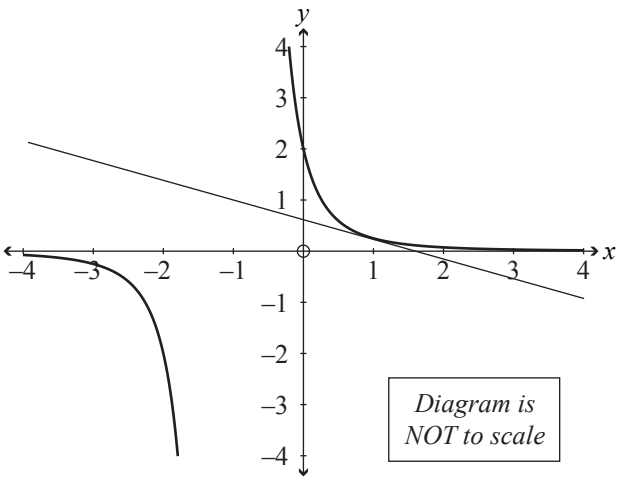
(a) Differentiate $y = \sqrt{3x - 2}$.

You do not need to simplify your answer.

(b) Find the rate of change of the function $f(t) = t^2 e^{2t}$ when $t = 1.5$.

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) The graph shows the curve $y = \dfrac{2}{(x+1)^3}$, along with the tangent to the curve drawn at $x = 1$.

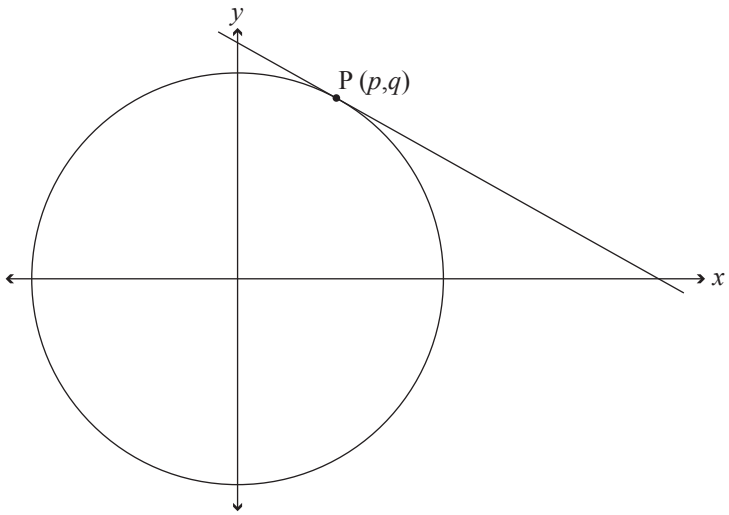


A second tangent to this curve is drawn which is parallel to the first tangent shown in the diagram.

Find the x-coordinate of the point where this second tangent touches the curve.

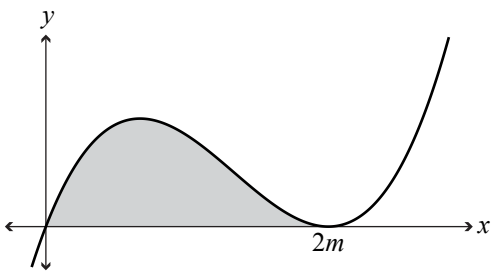
You must use calculus and show any derivatives that you need to find when solving this problem.

(d) The diagram below shows a tangent passing through the point P (p, q) which lies on the circle with parametric equations $x = 4\cos\theta$ and $y = 4\sin\theta$.

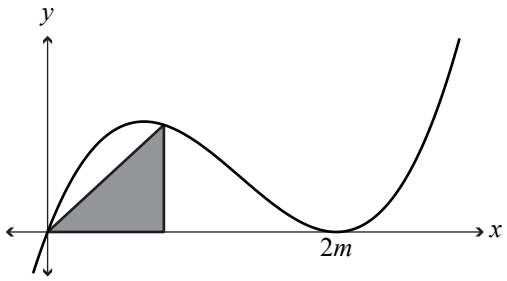


Show that the equation of the tangent line is $px + qy = p^2 + q^2$.

(e) The graph of $y = x(x - 2m)^2$, where $m > 0$, is shown.
The total shaded area between the curve and the x-axis from $x = 0$ to $x = 2m$ is given by $A = \frac{4m^4}{3}$.



A right-angled triangle is now constructed with one vertex at (0,0) and another on the curve $y = x(x - 2m)^2$, as shown below.



Show that the maximum area of such a triangle is $\frac{3}{8}$ of the total shaded area.

You must use calculus and show any derivatives that you need to find when solving this problem.
You do not have to prove that the area you have found is a maximum.

QUESTION TWO

(a) Differentiate $f(x) = \dfrac{x^2}{\cos x}$.

You do not need to simplify your answer.

(b) Find the gradient of the tangent to the curve $y = \cot(2x)$ at the point where $x = \dfrac{\pi}{12}$.

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) A curve is defined by the equation $f(x) = \dfrac{e^x}{x^2 + 2x}$.

Find the x-value(s) of any point(s) on the curve where the tangent to the curve is parallel to the x-axis.

You must use calculus and show any derivatives that you need to find when solving this problem.

(d) Find the x-value(s) of any points of inflection on the graph of the function $f(x) = 3x^2 \ln(x)$.

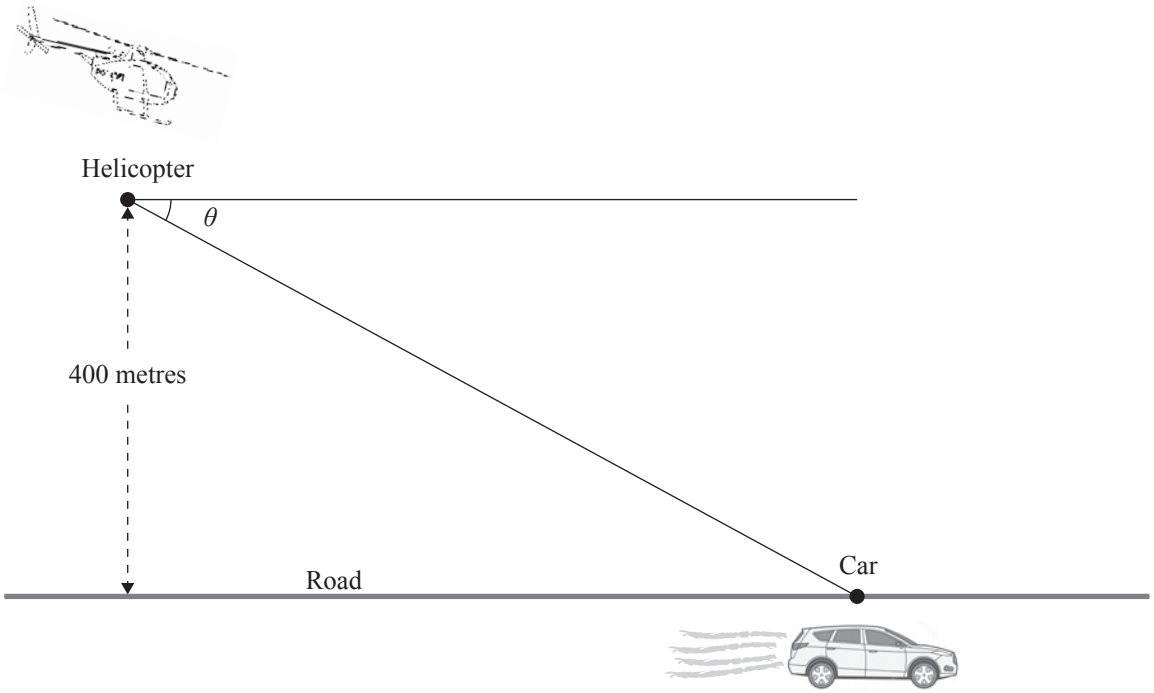
You can assume that your point(s) found are actually point(s) of inflection.

You must use calculus and show any derivatives that you need to find when solving this problem.

(e) A police helicopter is flying above a straight horizontal section of motorway chasing a speeding car.

The helicopter is flying at a constant speed of 72 m s^{-1} and at a constant height of 400 metres above the ground. The helicopter is attempting to catch up with the car.

When the direct distance from the helicopter to the car is 2500 metres, the angle of depression, θ, between the horizontal and the line of sight from the helicopter to the car is increasing at a rate of 0.002 rad s^{-1}.



Adapted from: https://animalia-life.club/qa/pictures/police-helicopter-drawing, https://www.freepik.com/premium-vector/car-sedan-suv-drawing-outlines-converted-objects_17981778.htm

Calculate the speed of the **car** at this instant.

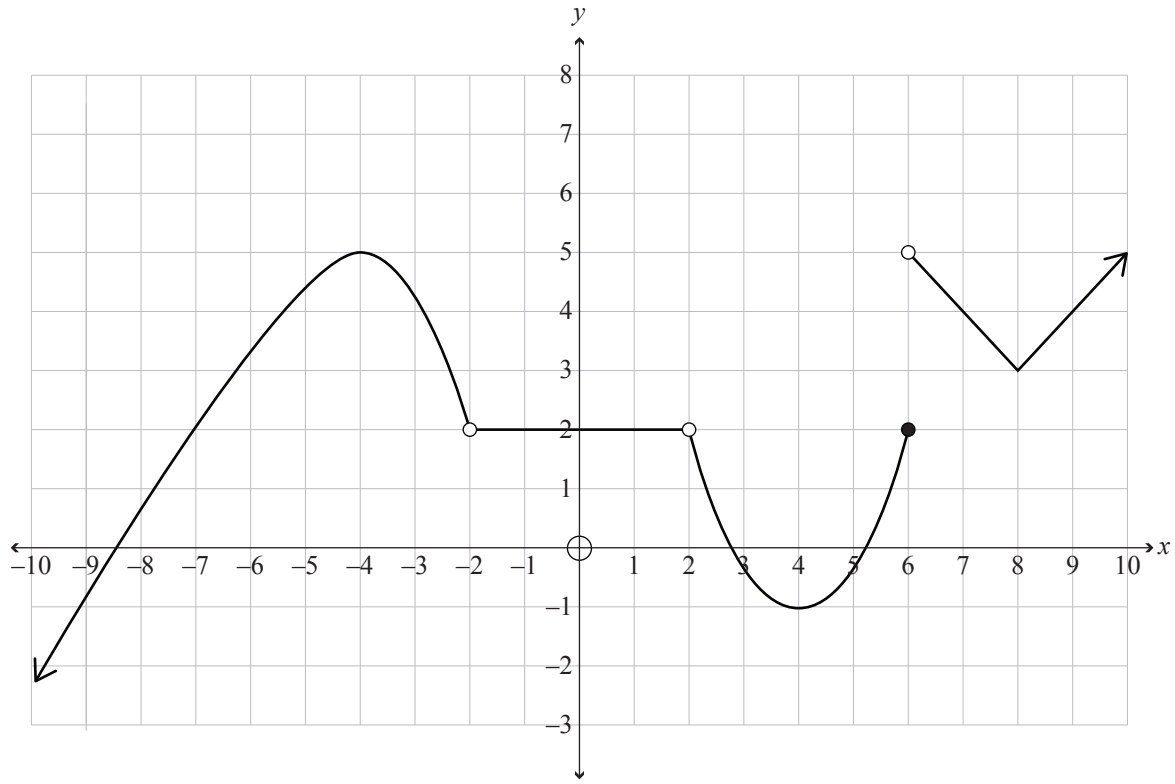
You must use calculus and show any derivatives that you need to find when solving this problem.

QUESTION THREE

(a) Differentiate $y = \ln(x^2 - x^4 + 1)$.

You do not need to simplify your answer.

(b) The graph below shows the function $y = f(x)$.



For the function above:

(i) Find the value(s) of x where $f(x)$ is continuous but not differentiable.

(ii) Find the value(s) of x where $f'(x) = 0$ and $f''(x) < 0$ are both true.

(iii) What is the value of $\lim_{x \to 6} f(x)$?

State clearly if the value does not exist.

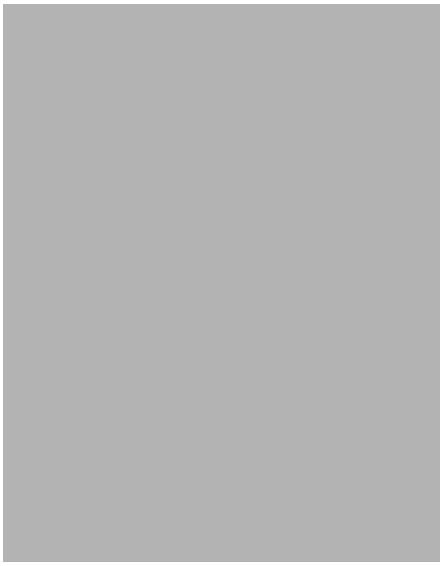
(c) Char goes for a ride on a Ferris wheel. As she rotates around, her position can be described by the pair of parametric equations :

$$x = 5\sqrt{2}\sin\left(\frac{\pi t}{5}\right) \text{ and } y = 10 - 5\sqrt{2}\cos\left(\frac{\pi t}{5}\right)$$

where t is time, in seconds, from the start of the ride.

Find the gradient of the normal to this curve at the point when $t = 6.25$ seconds, after the start of the ride.

You must use calculus and show any derivatives that you need to find when solving this problem.



Source: www.greenbaypressgazette.com/story/news/2019/06/27/bay-beachs-new-big-wheel-rolls-into-action-july-2/1582740001/

Question Three continues on the next page.

(d) Find the co-ordinates of any stationary points on the graph of the function $f(x)=\frac{1}{x}-\frac{2}{x^3}$, identifying their nature.

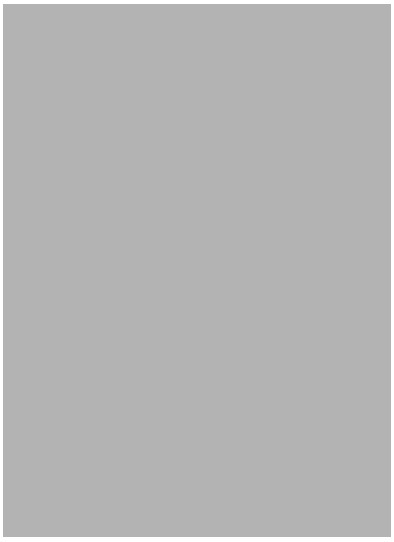
You must use calculus and show any derivatives that you need to find when solving this problem.

(e) A power line hangs between two poles.

The equation of the curve $y = f(x)$ that models the shape of the power line can be found by solving the differential equation:

$$a\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}$$

Use differentiation to verify that the function $y = \frac{a}{2}\left(\mathrm{e}^{\frac{x}{a}} + \mathrm{e}^{-\frac{x}{a}}\right)$ satisfies the above differential equation, where a is a positive constant.



Source: www.thelocalelectrician.com.au/power-lines-power-pole-who-is-responsible/

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER