

Assessment Schedule – 2021**Calculus: Apply integration methods in solving problems (91579)****Evidence Statement**

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
ONE (a)	$\frac{x^2}{6} + 3\ln x + c$	Correct integral + c not required.		
(b)(i)	$\frac{dy}{dx} = \frac{8}{x^3}$ $y = \int 8x^{-3} dx$ $y = -4x^{-2} + c$ $3 = -4 + c$ $c = 7$ $y = -4x^{-2} + 7$	Correct solution with correct integral.		
(ii)	$y = -4x^{-2} + 7$ $\text{Area} = \int_1^2 (-4x^{-2} + 7) dx$ $= \left[4x^{-1} + 7x \right]_1^2$ $= \left(\frac{4}{2} + 14 \right) - \left(\frac{4}{1} + 7 \right)$ $= 5$	Correct integral. (Limits not needed) CC: Integration of $y = -4x^{-2} + c$ From (b)(i).	Correct solution with correct integral. CC: Accept if correct solution for curve with any $+ c$ $y = -4x^{-2} + c$ From (b)(i).	
(c)	$a(t) = 2 - \sin 2t$ $v(t) = \int (2 - \sin 2t) dt$ $= 2t + \frac{\cos 2t}{2} + c$ $1 = 0 + \frac{\cos 0}{2} + c$ $c = 0.5$ $v(t) = 2t + \frac{\cos 2t}{2} + 0.5$ $s(t) = \int \left(2t + \frac{\cos 2t}{2} + 0.5 \right)$ $s(t) = t^2 + \frac{\sin 2t}{4} + 0.5t + k$ $t = 0, s = 3 \Rightarrow k = 3$ $s(5) = 5^2 + \frac{\sin 10}{4} + 0.5 \times 5 + 3$ $= 30.5 + \frac{1}{4} \sin 10$ $= 30.36$	Correct expression for $v(t)$.	Correct solution with correct integrals.	

(d)	$\frac{dV}{dt} = k\sqrt{V}$ $\int \frac{1}{\sqrt{V}} dv = \int k dt$ $\int V^{-0.5} dv = \int k dt$ $2\sqrt{V} = kt + c$ $T = 6, V = 400, 40 = 6k + c$ $T = 10, V = 256, 32 = 10k + c$ $k = -2 \quad c = 52$ $\therefore 2\sqrt{V} = -2t + 52$ $t = 0$ $2\sqrt{V} = 52$ $\sqrt{V} = 26$ $V = 676 \text{ (litres)}$	Correct general solution to DE. + c not required.	Correct general solution to DE plus correct values of c and k .	Correct solution (units not required)
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NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
TWO (a)	$\frac{e^{4x}}{4} + \frac{8}{3}x^{\frac{3}{2}} + c$	Correct integral + c not required.		
(b)	14	Correct solution.		
(c)	$\int_0^{\frac{\pi}{8}} \sin 6x \cdot \sin 2x \, dx = \frac{1}{2} \int_0^{\frac{\pi}{8}} (\cos 4x - \cos 8x) \, dx$ $= \frac{1}{2} \left[\frac{\sin 4x}{4} - \frac{\sin 8x}{8} \right]_0^{\frac{\pi}{8}}$ $= \frac{1}{2} \left[\left(\frac{\sin \frac{\pi}{2}}{4} - \frac{\sin \pi}{8} \right) - 0 \right]$ $= \frac{1}{8}$	Correct integration. (Limits not needed)	Correct solution with correct integration.	
(d)	$\int_2^k \frac{6}{3x-4} \, dx = \left[2 \ln(3x-4) \right]_2^k$ $= 2 \ln(3k-4) - 2 \ln(2) = 4$ $2 \ln(3k-4) = 4 + 2 \ln(2)$ $\ln(3k-4) = 2 + \ln(2)$ $3k-4 = e^{2+\ln 2}$ $3k-4 = 2e^2$ $k = \frac{2e^2+4}{3}$ $k = 6.26$	Correct integration. (Limits not needed)	Correct solution with correct integration. Accept $k = \frac{2e^{2+\ln 2} + 4}{3}$.	
(e)	$\frac{dy}{dx} = \frac{2}{ye^{0.5x}}$ $\int y \, dy = \int 2e^{-0.5x} \, dx$ $\frac{y^2}{2} = -4e^{-0.5x} + c$ $x = 0 \text{ and } y = 1$ $\frac{1}{2} = -4 + c$ $c = 4.5$ $\frac{y^2}{2} = -4e^{-0.5x} + 4.5$ $\text{At } x = 3:$ $\frac{y^2}{2} = -4e^{-1.5} + 4.5$ $y = 2.686$ $\text{Distance} = 5.372$	Correct integration with correct rearrangement of both sides. + c not required.	Correct solution of DE.	Correct solution with correct integrations shown. E7 $y = \pm 2.686$

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
THREE (a)	$\int (x + \sqrt{x})^2 dx$ $= \int (x^2 + 2x\sqrt{x} + x) dx$ $= \int \left(x^2 + 2x^{\frac{3}{2}} + x \right) dx$ $= \frac{x^3}{3} + \frac{4}{5}x^{\frac{5}{2}} + \frac{x^2}{2} + c$	Correct integral + c not required.		
(b)	2.6	Correct solution.		
(c)	$\frac{dy}{dx} = \frac{\sec^2 2x}{y}$ $\int y dy = \int \sec^2 2x dx$ $\frac{y^2}{2} = \frac{\tan 2x}{2} + c$ $x = \frac{3\pi}{4}, y = 2$ $\frac{2^2}{2} = \frac{\tan \frac{3\pi}{4}}{2} + c$ $2 = \frac{-1}{2} + c$ $c = 2.5$ $\frac{y^2}{2} = \frac{\tan 2x}{2} + 2.5$ $x = \pi$ $\frac{y^2}{2} = \frac{\tan 2\pi}{2} + 2.5$ $\frac{y^2}{2} = 2.5$ $y = \pm\sqrt{5} = \pm 2.236$	Correct integration after correct separation of both sides. (+ c not required)	Correct solution with correct integration after correct separation of both sides ($\pm$ not required)	
(d)	$y = \frac{3x-2}{x+2}$ $y = 1 \Rightarrow x = 2 \quad y = 2 \Rightarrow x = 6$ $\frac{3x-2}{x+2} = 3 - \frac{8}{x+2}$ $\int_2^6 \left(3 - \frac{8}{x+2} \right) dx$ $= \left[3x - 8 \ln(x+2) \right]_2^6$ $= (18 - 8 \ln 8) - (6 - 8 \ln 4)$ $= 12 + 8 \ln 0.5 \text{ (or } 12 - 8 \ln 2)$ $= 6.455$	Correct integration. (Limits not needed.)	Correct solution with correct integration.	

(e)	$(ke^x)^2 = k$ $k^2 e^{2x} = k$ $e^{2x} = \frac{1}{k}$ $x = \frac{\ln \frac{1}{k}}{2}$ $\text{Area} = k^2 \int_{\frac{\ln \frac{1}{k}}{2}}^0 e^{2x} dx - \int_{\frac{\ln \frac{1}{k}}{2}}^0 k dx$ $= \frac{k^2}{2} \left[e^{2x} \right]_{\frac{\ln \frac{1}{k}}{2}}^0 - \left[kx \right]_{\frac{\ln \frac{1}{k}}{2}}^0$ $= \frac{k^2}{2} \left(1 - e^{2 \times \frac{\ln \frac{1}{k}}{2}} \right) - \left(0 - k \cdot \frac{\ln \frac{1}{k}}{2} \right)$ $= \frac{k^2}{2} \left(1 - \frac{1}{k} \right) + \frac{k}{2} \cdot \ln \frac{1}{k}$ $= \frac{k^2}{2} - \frac{k}{2} + \frac{k}{2} \cdot \ln \frac{1}{k}$ $= \frac{k}{2} \left(k - 1 + \ln \frac{1}{k} \right)$ OR $\text{Area} = \int_{\frac{\ln \frac{1}{k}}{2}}^0 (k^2 e^{2x} - k) dx$ $= \left[\frac{k^2}{2} e^{2x} - kx \right]_{\frac{\ln \frac{1}{k}}{2}}^0$ $= \left(\frac{k^2}{2} - 0 \right) - \left(\left[\frac{k^2}{2} e^{\ln \frac{1}{k}} - \frac{k^2}{2} \ln \frac{1}{k} \right] \right)$ $= \frac{k^2}{2} - \frac{k}{2} + \frac{k}{2} \ln \frac{1}{k}$ $= \frac{k}{2} \left(k - 1 + \ln \frac{1}{k} \right)$	Correct integration. (Limits not needed.)	Correct integration with correct limits substituted in.	Correct solution with steps clearly shown.
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No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 6	7 – 13	14 – 19	20 – 24