

Assessment Schedule – 2022**Calculus: Apply integration methods in solving problems (91579)****Evidence Statement**

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
ONE (a)	$4 \ln x - \tan x + c$	Correct integral.		
(b)	6.4	Correct solution.		
(c)	$\int_0^{\frac{\pi}{4}} \sin^2(2x) dx$ $= \int_0^{\frac{\pi}{4}} \left(\frac{1}{2} - \frac{1}{2} \cos(4x) \right) dx$ $= \left[\frac{x}{2} - \frac{1}{8} \sin 4x \right]_0^{\frac{\pi}{4}}$ $= \left(\frac{\pi}{8} - \frac{1}{8} \sin \pi \right) - (0 - 0)$ $= \frac{\pi}{8}$	Correct integral.	Correct solution with correct integral. Accept 0.393.	
(d)	$y = \frac{4}{\sqrt{3x-2}} = 4(3x-2)^{-\frac{1}{2}}$ $\text{Area} = \int_1^k 4(3x-2)^{-\frac{1}{2}} dx$ $= \left[\frac{8}{3} (3x-2)^{\frac{1}{2}} \right]_1^k$ $= \frac{8}{3} \sqrt{3k-2} - \frac{8}{3} \times 1 = 8$ $\sqrt{3k-2} - 1 = 3$ $\sqrt{3k-2} = 4$ $3k-2 = 16$ $k = 6$	Correct integral.	Correct solution with correct integral.	

(e)	Limits of integration $(e^x)^2 = 3e^x + 10$ $(e^x)^2 - 3e^x - 10 = 0$ $(e^x - 5)(e^x + 2) = 0$ $e^x = 5 \quad \text{or} \quad e^x = -2 \quad \text{no}$ $x = \ln 5$ $\text{Area} = \int_0^{\ln 5} (3e^x + 10) dx - \int_0^{\ln 5} e^{2x} dx$ $= \int_0^{\ln 5} (3e^x + 10 - e^{2x}) dx$ $= \left[3e^x + 10x - \frac{e^{2x}}{2} \right]_0^{\ln 5}$ $= \left(3e^{\ln 5} + 10\ln 5 - \frac{e^{2\ln 5}}{2} \right) - \left(3 + 0 - \frac{1}{2} \right)$ $= (15 + 10\ln 5 - 12.5) - 2.5$ $= 10\ln 5$	Correct integral expression for area with correct limits and e^{2x} .	Correct integration.	Correct solution with correct integration.
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NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
TWO (a)	$\frac{e^{3x}}{3} - \frac{2}{3}x^{\frac{3}{2}} + c$	Correct integral.		
(b)	$\int_1^k \frac{2}{\sqrt{x}} dx = 8$ $\int_1^k 2x^{-\frac{1}{2}} dx = 8$ $\left[4\sqrt{x}\right]_1^k = 8$ $4\sqrt{k} - 4 = 8$ $4\sqrt{k} = 12$ $\sqrt{k} = 3$ $k = 9$	Correct solution with correct integral.		
(c)	$\frac{dy}{dx} = \frac{1}{3y^2(x-1)}$ $\int 3y^2 dy = \int \frac{1}{x-1} dx$ $y^3 = \ln x-1 + c$ $x = 2, y = -1$ $-1 = \ln(1) + c$ $c = -1$ When $y = 1$ $1 = \ln(x-1) - 1$ $\ln(x-1) = 2$ $x = e^2 + 1 (= 8.389)$	Correct integration.	Correct solution with correct integral.	
(d)	$a(t) = 0.9e^{0.3t}$ $v(t) = 3e^{0.3t} + c$ When $t = 2, v = 10$ $10 = 3e^{0.6} + c$ $c = 4.534$ $v(t) = 3e^{0.3t} + 4.534$ Distance travelled in 5th second of motion $= \int_4^5 (3e^{0.3t} + 4.534) dt$ $= \left[10e^{0.3t} + 4.534t\right]_4^5$ $= (10e^{1.5} + 4.534 \times 5) - (10e^{1.2} + 4.534 \times 4)$ $= 16.1497 \text{ m}$	Correct equation for v .	Correct solution with correct integrals.	

(e)	$\frac{dh}{dt} = \frac{-1}{4} \sqrt{(h-6)^3}$ $\int (h-6)^{\frac{-3}{2}} dh = \int \frac{-1}{4} dt$ $-2(h-6)^{\frac{-1}{2}} = \frac{-t}{4} + c$ $(h-6)^{\frac{-1}{2}} = \frac{t}{8} + k$ $\frac{1}{\sqrt{h-6}} = \frac{t}{8} + k$ When $t = 0, h = 150$ $\frac{1}{\sqrt{144}} = k$ $k = \frac{1}{12}$ $\frac{1}{\sqrt{h-6}} = \frac{t}{8} + \frac{1}{12}$ $h = 15$ $\frac{1}{\sqrt{9}} = \frac{t}{8} + \frac{1}{12}$ $\frac{t}{8} = \frac{1}{4}$ $t = 2$	Correct integral wrt h .	Correct solution of DE.	Correct solution of problem with all integrals correct.
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NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
THREE (a)	$\frac{1}{8}(2x+5)^4 + c$	Correct integral.		
(b)	8.92	Correct solution.		
(c)	$\int_5^8 \frac{4x-5}{x-3} dx$ $\frac{4x-5}{x-3} = 4 + \frac{7}{x-3}$ $\int_5^8 \frac{4x-5}{x-3} dx = \int_5^8 \left(4 + \frac{7}{x-3} \right) dx$ $= [4x + 7 \ln x-3]_5^8$ $= (32 + 7 \ln 5) - (20 + 7 \ln 2)$ $= 18.41$	Correct integral.	Correct solution with correct integral.	
(d)	Limits of integration $x = x + \cos x$ $\cos x = 0$ $x = \frac{-\pi}{2}, \frac{\pi}{2}$ $\text{Area} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x + \cos x) dx - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x dx$ $= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos x) dx$ $= [\sin x]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$ $= 1 - -1$ $= 2$	Correct integral.	Correct solution with correct integral.	
(e)	Area of rectangle $= 2k \times \frac{2}{k} = 4$ Area under curve $= \int_k^{3k} \frac{2}{x} dx$ $= [2 \ln x]_k^{3k}$ $= 2 \ln 3k - 2 \ln k$ $= \ln 9k^2 - \ln k^2$ $= \ln \left(\frac{9k^2}{k^2} \right)$ $= \ln 9$ Shaded area $= 4 - \ln 9$ $a = 4, b = -1, \text{ and } c = 9.$	Correct integration.	Correct area under curve with correct integration.	Correct solution. Accept $4 - \ln 9$. Accept $4 - 2 \ln 3$.

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No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 7	8 – 14	15 – 20	21 – 24