Assessment Schedule – 2023

Calculus: Apply integration methods in solving problems (91579)

Evidence Statement

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
ONE (a)	$= \frac{3x^2}{2} + 2x + \frac{1}{3}\ln(3x+2) + c$	• Correct integral $+c$ not required.		
(b)	$d = \int \sec^2 t \, dt$ $\Rightarrow d = \tan t + c$ $t = 0$ and $d = 3$ give $3 = \tan 0 + c$ $\Rightarrow c = 3$ i.e. $d = \tan t + 3$ $t = \frac{\pi}{4} \Rightarrow d = \tan\frac{\pi}{4} + 3$ $d = 1 + 3 = 4$ km	• Correct solution, with evidence of correct integration.		
(c)	For point of intersection between the two curves, solve $\sqrt{x} = \frac{x^2}{8}$, Giving $x = 0$ and $x = 4$. Area $= \int_0^4 \left(\sqrt{x} - \frac{x^2}{8}\right) dx$ $= \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{x^3}{3 \times 8}\right]_0^4$ $= \left[\frac{16}{3} - \frac{64}{24}\right] - [0] = \frac{8}{3}$ units^2	• Correct integral.	• Correct solution, with evidence of correct integration.	
(d)	$\frac{dy}{dx} = y(2x - 3x^2)$ Separating the variables gives: $\int \frac{1}{y} dy = \int (2x - 3x^2) dx$ $\ln y = x^2 - x^3 + c$ #(1) $x = 2$ and $y = 1$ gives: $\ln 1 = 4 - 8 + c \Rightarrow c = 4$ i.e. $\ln y = x^2 - x^3 + 4$ $x = 1 \Rightarrow \ln y = 1 - 1 + 4$ $\Rightarrow \ln y = 4$ $y = e^4 = 54.6$	• Reaching stage #(1) of the solution.	• Correct solution, with evidence of correct integration.	

(e)	Intersection points are (1,3) and (10,0). Area of Triangle A $=\frac{1}{2}\times 1\times 3=1.5$ (**or** by integration) Area of $\text{B}=\int_1^{10}\sqrt{10-x}\,\mathrm{d}x$ $=\int_1^{10}(10-x)^{\frac{1}{2}}\,\mathrm{d}x$ $=\left[-\frac{2}{3}(10-x)^{\frac{3}{2}}\right]_1^{10}$ $=-\frac{2}{3}\left[0-27\right]=18$ Area of $\text{C}=\int_0^{10}-\sin\frac{\pi x}{10}\,\mathrm{d}x$ $=\left[\frac{10}{\pi}\cos\frac{\pi x}{10}\right]_0^{10}$ $=\frac{10}{\pi}\left[\cos\pi-\cos 0\right]$ $=\frac{10}{\pi}\left[-1-1\right]$ $=-\frac{20}{\pi}$ i.e. Actual area will be $\frac{20}{\pi}$. Total area $1.5+18+\frac{20}{\pi}$ $=25.866$ units^2	• Correct integration of one of the expressions.	Correct evaluation of Area B and C.	E 7 Solution with one minor error. E 8 Correct area, with evidence of correct integrations.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
TWO (a)	$\int 4\mathrm{e}^{2x-1}\,\mathrm{d}x = 2\mathrm{e}^{2x-1} + c$	• Correct integral, +c not required.		
(b)	$y = \int (4x+1)^{-\frac{1}{2}}\,\mathrm{d}x$ $y = \frac{1}{4}\frac{(4x+1)^{\frac{1}{2}}}{\frac{1}{2}} + c$ $y = \frac{1}{2}(4x+1)^{\frac{1}{2}} + c$ $x = 6$ and $y = 7.5$ gives: $7.5 = \frac{1}{2} \times 25^{\frac{1}{2}} + c \Rightarrow c = 5$ i.e. $y = \frac{1}{2}(4x+1)^{\frac{1}{2}} + 5$	• Correct solution, with evidence of correct integration.		
(c)	$\int_2^k \left(3 + \frac{6}{2x-3}\right)\mathrm{d}x = 3k$ $\Rightarrow \left[3x + 3\ln(2x-3)\right]_2^k = 3k$ $\Rightarrow 3k + 3\ln(2k-3) - 6 = 3k$ $\Rightarrow 3\ln(2k-3) = 6$ $\Rightarrow \ln(2k-3) = 2$ $\Rightarrow 2k - 3 = \mathrm{e}^2$ $\Rightarrow k = \frac{\mathrm{e}^2 + 3}{2} = 5.1945$	• Correct integral.	• Correct solution, with evidence of correct integration.	
(d)	$= -\frac{1}{2}\int \frac{-2\cos 2x - 2\sin 2x}{\cos 2x - \sin 2x}\,\mathrm{d}x$ $= -\frac{1}{2} \times \ln(\cos 2x - \sin 2x) + c$	• Integral, but with missing negative sign.	• Correct integral.	

(e)	$\frac{dv}{dt} = -kv^2$ $\int \frac{1}{v^2}\,dv = \int -k\,dt$ $\int v^{-2}\,dv = \int -k\,dt$ $\frac{v^{-1}}{-1} = -kt + c$ #(1) $\frac{-1}{v} = -kt + c$ $\frac{1}{v} = kt + d$ $t = 1$ and $v = p$ gives: $\frac{1}{p} = k + d$ eq(1) $t = 2$ and $v = \frac{4}{5}p$ gives: $\frac{1}{\frac{4}{5}p} = 2k + d$ $\frac{5}{4p} = 2k + d$ eq(2) $\text{eq}(2) - \text{eq}(1) \Rightarrow \frac{5}{4p} - \frac{1}{p} = k$ i.e. $k = \frac{1}{4p}$ Then eq(1) gives $\frac{1}{p} = \frac{1}{4p} + d$ $d = \frac{3}{4p}$ i.e. $\frac{1}{v} = \frac{1}{4p}t + \frac{3}{4p}$ $t = 0$ gives $\frac{1}{v} = 0 + \frac{3}{4p}$ #(2) i.e. $v = \frac{4}{3}p$ i.e. The container had $\frac{4}{3}p$ litres of chocolate initially.	• Reaching stage #(1) of the solution.	• Finding the value of k, in terms of p. OR Finding the value of d, in terms of p.	E 7 Reaching stage #(2) of the solution. OR Solution with one minor error E 8 Correct volume of chocolate initially, with evidence of correct integrations.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
THREE (a)	$\text{Area} \approx \frac{1}{3}\times 3\times\left[10+14+4\left(13+15\right)+2\left(16\right)\right]$ $=1\times\left[24+112+32\right]$ $=168 \text{ m}^2$ *Units not required.*	• Correct solution.		
(b)	$=\int\left(1-\frac{3}{x^{\frac{1}{2}}}\right)\mathrm{d}x$ $=\int\left(1-3x^{-\frac{1}{2}}\right)\mathrm{d}x$ $=x-\frac{3x^{\frac{1}{2}}}{\frac{1}{2}}+c$ $=x-6x^{\frac{1}{2}}+c$	• Correct integral, +c not required.		
(c)	$=\int_0^{\frac{\pi}{3}}5\sin 3x\sin x\,\mathrm{d}x$ $=\frac{5}{2}\int_0^{\frac{\pi}{3}}2\sin 3x\sin x\,\mathrm{d}x$ $=\frac{5}{2}\int_0^{\frac{\pi}{3}}\cos 2x-\cos 4x\,\mathrm{d}x$ $=\frac{5}{2}\left[\frac{1}{2}\sin 2x-\frac{1}{4}\sin 4x\right]_0^{\frac{\pi}{3}}$ $=\frac{5}{2}\left[0.6495-0\right]$ $=\frac{5}{2}\times 0.6495$ $=1.6238 \text{ units}^2$ or $\frac{15\sqrt{3}}{16}$	• Correct integral.	• Correct area, with evidence of correct integration.	
(d)	$v=\int\frac{e^{2t}}{4e^{2t}-3}\mathrm{d}t$ $v=\frac{1}{8}\int\frac{8e^{2t}}{4e^{2t}-3}\mathrm{d}t$ $v=\frac{1}{8}\ln\left(4e^{2t}-3\right)+c$ $t=0$ and $v=5$ gives $5=\frac{1}{8}\ln 1+c\Rightarrow c=5$ i.e. $v=\frac{1}{8}\ln\left(4e^{2t}-3\right)+5$ $t=4$ gives $v=6.1732 \text{ m s}^{-1}$	• Correct equation for v, with evidence of correct integration.	• Correct solution, with evidence of correct integration.	

(e)	$\int \frac{1+y}{1-y^2}\mathrm{d}y = -\int \frac{1-x}{1-x^2}\mathrm{d}x$ $\int \frac{1+y}{(1-y)(1+y)}\mathrm{d}y = -\int \frac{1-x}{(1-x)(1+x)}\mathrm{d}x$ $\int \frac{1}{(1-y)}\mathrm{d}y = -\int \frac{1}{(1+x)}\mathrm{d}x$ $-\ln(1-y) = -\ln(1+x) + c$ #(1) $y = 0$ and $x = 2$ gives: $-\ln 1 = -\ln 3 + c$ $\Rightarrow c = \ln 3 = 1.09863$ i.e. $-\ln(1-y) = -\ln(1+x) + \ln 3$ #(2) $\ln(1-y) = \ln(1+x) - \ln 3$ $\ln(1-y) = \ln\left(\frac{1+x}{3}\right)$ $1 - y = \frac{1+x}{3}$ #(3) $1 - \frac{1}{3} - \frac{x}{3} = y$ $y = \frac{2-x}{3}$ $x = 6$ gives $y = \frac{-4}{3}$ OR Alternative solution from #(2) onwards $-\ln(1-y) = -\ln(1+x) + 1.09863$ $\ln(1-y) = \ln(1+x) - 1.09863$ $\ln(1-y) - \ln(1+x) = -1.09863$ $\ln\left(\frac{1-y}{1+x}\right) = -1.09863$ $\frac{1-y}{1+x} = \mathrm{e}^{-1.09863}$ #(3) $\frac{1-y}{1+x} = \frac{1}{3}$ $y = \frac{2-x}{3}$ $x = 6$ gives $y = \frac{-4}{3}$	• Correct integral for either of the two integrals at stage #(1) of the solution.	• Correct solution of the differential equation, i.e. stage #(2) of the solution.	E 7 Solution with one minor error. E 8 Correct solution, with evidence of correct integrations.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 6	7 – 12	13 – 8	19 – 24