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SUPERVISOR'S USE ONLY

Level 3 Calculus, 2015

91579 Apply integration methods in solving problems

2.00 p.m. Wednesday 25 November 2015
 Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply integration methods in solving problems.	Apply integration methods, using relational thinking, in solving problems.	Apply integration methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

ASSESSOR'S USE ONLY

QUESTION ONEASSESSOR'S
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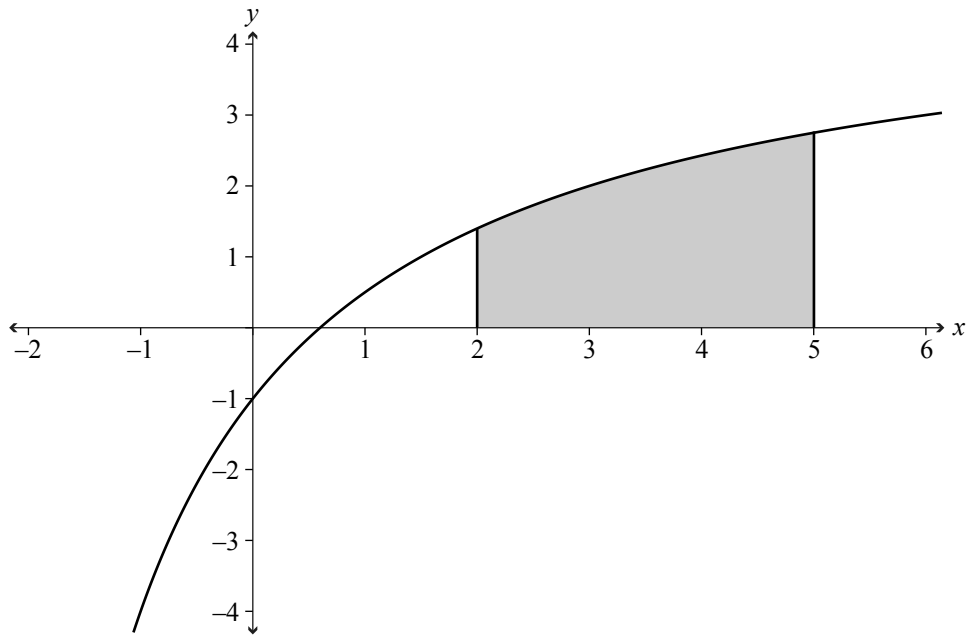
- (a) Find $\int (\sqrt{x} + 6 \cos 2x) dx$.

- (b) Solve the differential equation $\frac{dy}{dx} = \frac{2}{x}$, given that when $x = 1$, $y = 3$.

- (c) If $\frac{dy}{dx} = \frac{e^{2x}}{4y}$ and $y = 4$ when $x = 0$, find the value of y when $x = 2$.

- (d) Use integration to find the area enclosed between the curve $y = \frac{5x-3}{x+3}$ and the lines $y = 0$, $x = 2$ and $x = 5$.

The area is shown shaded in the diagram below.



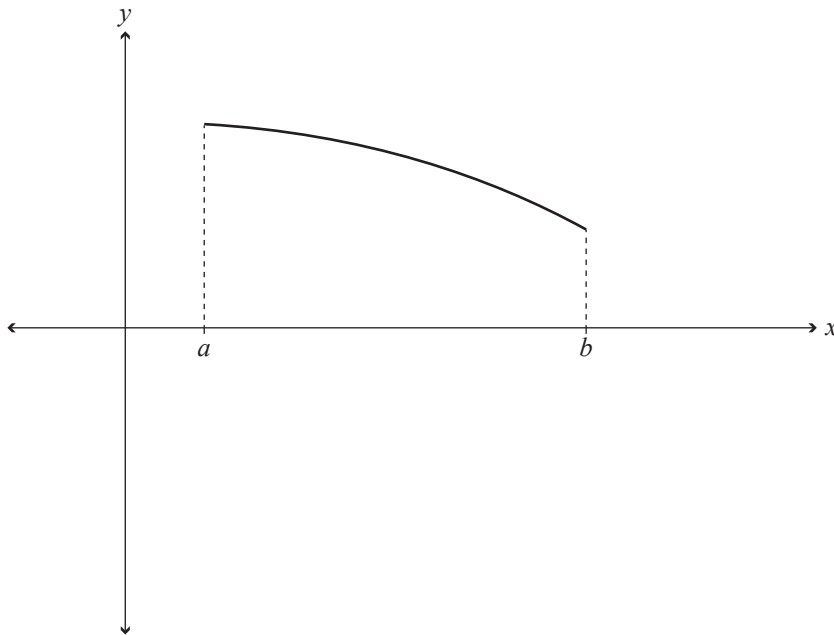
Show your working.

You must use calculus and give the results of any integration needed to solve this problem.

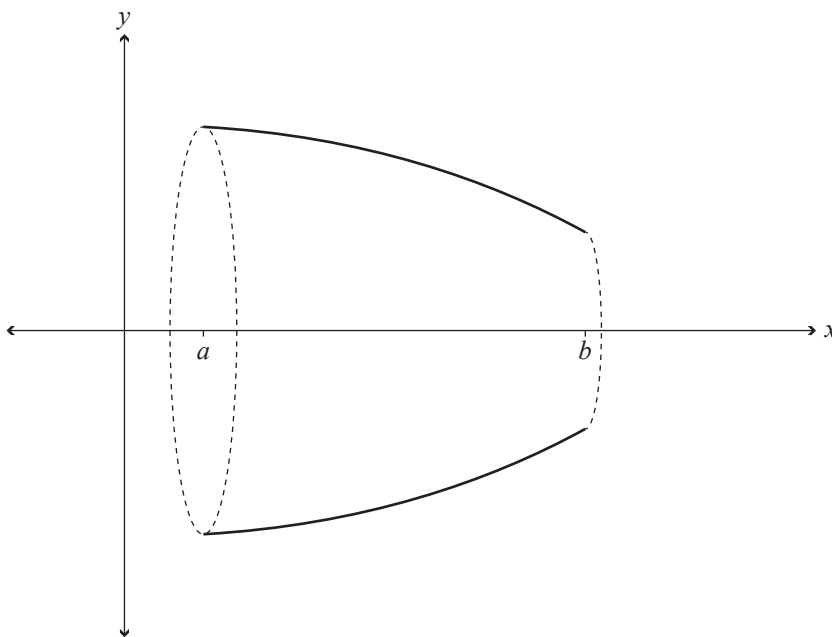
(e)

ASSESSOR'S
USE ONLY

Consider the curve defined by the function $y = f(x)$, bounded by $x = a$ and $x = b$.



This portion of the curve is rotated around the x-axis, as shown below.



The volume created by this rotation is given by the formula

$$\text{Volume} = \pi \int_a^b (f(x))^2 dx$$

You must use calculus and give the results of any integration needed to solve this problem.

QUESTION TWO

ASSESSOR'S
USE ONLY

(a) Find $\int \left(3 - \frac{5}{x^2} \right) dx$.

- (b) Use the values given in the table below to find an approximation to $\int_1^{2.5} f(x) dx$, using the Trapezium Rule.

x	1	1.25	1.5	1.75	2	2.25	2.5
$f(x)$	0.3	0.7	1.65	1.9	2.35	1.7	1.1

- (c) An object originally moving at a constant velocity suddenly starts to accelerate. From the start of the object's acceleration the motion of the object can be modelled by the differential equation

$$\frac{dv}{dt} = \frac{50t^2 - 80\sqrt{t}}{5\sqrt{t}} \text{ for } 0 \leq t \leq 20$$

where v is the velocity of the object in m s^{-1}

and t is the time in seconds after the object starts to accelerate.

If the original velocity of the object was 6 m s^{-1} , find the velocity of the object when $t = 4$.

You must use calculus and give the results of any integration needed to solve this problem.

- (d) In the town of Clarkeville, the rate at which the population, P , of the town changes at any instant is proportional to the population of the town at that instant.

- (i) Write a differential equation which models this situation.

- (ii) At the start of 2000, the population of the town was 12 000.

At the start of 2010, the population of the town was 16 000.

Solve the differential equation in (i) to find the population the town will have at the start of 2025.

You must use calculus and give the results of any integration needed to solve this problem.

-
- A Cartesian coordinate system showing the function $y = \frac{1}{x}$ (a hyperbola with branches in the first and third quadrants) and the line $y = x$. The line passes through the origin. The region in the first quadrant bounded by the curve, the line, and the x-axis from $x = k$ to the intersection point is shaded gray.

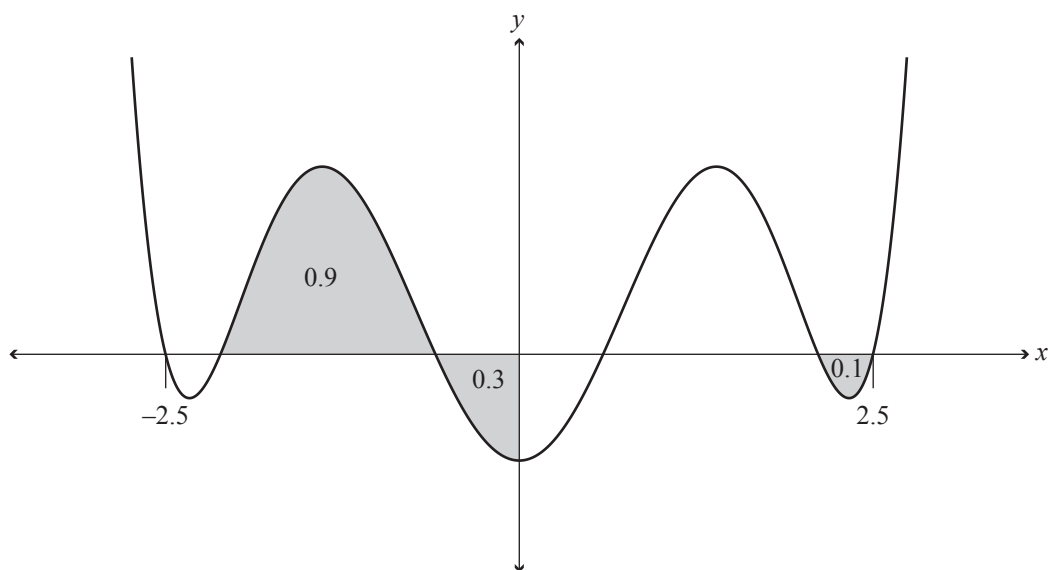
Find the value of k .

QUESTION THREE

ASSESSOR'S
USE ONLY

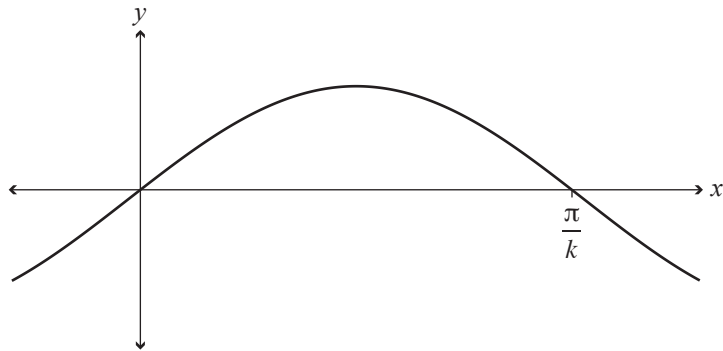
(a) Find $\int \left((x+4)^2 + 8e^{4x} \right) dx$.

- (b) The graph of the function $y = f(x)$ below is symmetrical about the y -axis.
The areas of the shaded regions are given.



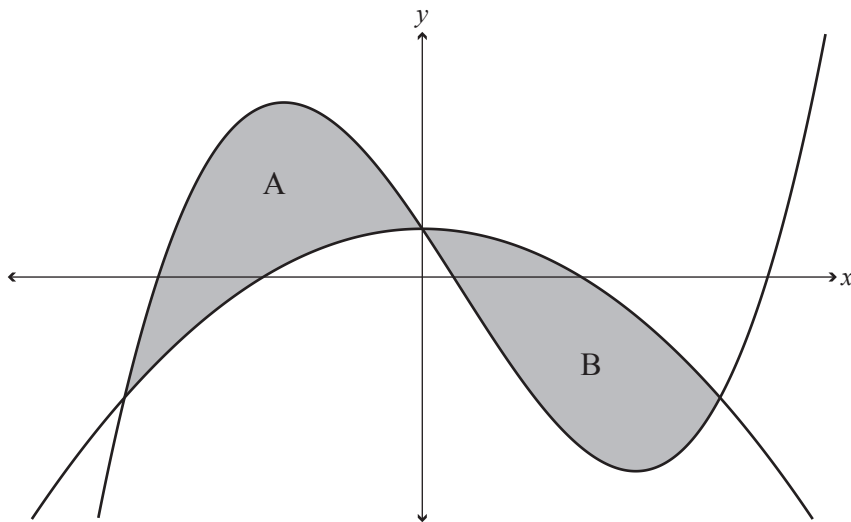
Find $\int_{-2.5}^{2.5} f(x) dx$.

- (c) Find an expression in terms of k for the area bounded by the function $y = \sin kx$ and the x -axis, between $x = 0$ and $x = \frac{\pi}{k}$.



You must use calculus and give the results of any integration needed to solve this problem.

- The graphs intersect and create two closed regions, A and B.



Show that these two regions have the same area.

You must use calculus and give the results of any integration needed to solve this problem.

- You must use calculus and give the results of any integration needed to solve this problem.*

Extra paper if required.
Write the question number(s) if applicable.

QUESTION
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