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## NZEST SCHOLARSHIP EXAMINATION

# 1995 EXAMINER'S REPORT AND SOLUTIONS

# MATHEMATICS WITH CALCULUS

# NZEST SCHOLARSHIP EXAMINATION 1995

# MATHEMATICS WITH CALCULUS

The general plan of the examination this year was as for 1994: 6 questions with the possibility of full marks being obtained from the answers to only 5 of them. There were 625 candidates. The range was 0 to 100, with upper quartile 69, median 53 and lower quartile 40.

The following table shows the numbers of marks in the ranges indicated:

| 0-9 | 10-19 | 20-29 | 30-39 | 40-49 | 50-59 | 60-69 | 70-79 | 80-89 | 90-100 |
|---|---|---|---|---|---|---|---|---|---|
| 10 | 20 | 55 | 68 | 104 | 119 | 93 | 63 | 50 | 43 |

The following table shows the number of candidates attempting each question and the mean mark:

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 604 | 612 | 610 | 605 | 570 | 500 |
| 12 | 8 | 11 | 9 | 11 | 4 |

General comments on the marking philosophy:

The markers were instructed to allocate marks in one of three grades: easy, medium and hard. "Easy" meant that the mark should be given when the answer was on the right track, "medium" when the answer was almost correct and "hard" only when the answer was error-free. The questions did not have an even distribution of these three grades of marks. This was intended to reflect the differing nature of the questions.

Question 1

The 5 most common marks in this question were, in descending order of frequency: 20; 19; 9; 17 and 12. The marker observed that students seemed either to be very comfortable with complex numbers or else found the idea of a number with two parts too hard to accept. Students sometimes tried to work too quickly and when they didn't it was sometimes apparent that they were confused by the "sic" notation. Many students appeared to have a tenuous grasp of de Moivre's theorem. The less able students did not like dealing with $\pi$ and converted their answers to degrees, which was accepted, or used calculators to convert to a numerical answer in radians. An inductive proof was sometimes used for part (c), and those few who succeeded in this were given full marks even though they were asked specifically to deduce the answer from (b): the inductive proof is much more complicated! In part (d) many found the general solution and then particularised to the cases asked for; symmetry was also used successfully. An all too frequent error in proofs was to multiply one side of an equation by i or some multiple so that the right answer was obtained; it would seem that teachers may need to check this bad habit. Another way of deriving a "proof" was to assume the answer which is sought and eventually reach the point where they deduce that 0=0.

Some common errors included:

- assuming that $i^n = 1$ always, which appeared to help in part (c);
- a new version of de Moivre's theorem where $r = i$;
- the use of implication signs where equal signs are called for.

Question 2

The 5 most common marks in this question were, in descending order of frequency: 6; 4; 7; 3 and 10 and 12 equally. This question contained many proofs which challenged both the candidates and the markers. The conclusion which the candidates were asked to draw is rather profound! Parts (a), (b), (d) and (h) were well done. It was usually recognised in part (c) that extra terms were needed but there was frequently no observation that these terms are positive. Part (e)(i) was well-done by those who recognised the geometric series but those who tried to use induction found the question hard. Part (e)(ii) was difficult to mark because it was common to multiply both sides by $(n-1)!$ and then try to show that the result on the right approaches 1 without justifying the required conclusion. Part (f) was also found to be difficult. The final conclusion was also found to be hard to explain, with a common assumption that e is irrational so couldn't be written as a fraction!

Question 3

The 5 most common marks in this question were, in descending order of frequency: 18; 6 and 17 equally; and 7 and 9 equally. Broken up as it is into many small parts, this question provided the opportunity for candidates to pick up a number of easy marks yet only the top candidates were able to earn more than 75% on the question. The parts which were well done were (a), (b), (d) and (e)(i). A large number of candidates did not make X or Y the subject of the equations in parts (c)(i) and (ii). The answers in (c)(iii) were well done, with candidates showing good insight and providing logical explanations which were well set out and generally accurate. The results in (e)(i) and (ii) were also well done with the results being obtained by a variety of methods. Frequently in (e) even average candidates scored marks at both the beginning and the end of the question though they were sometimes lost in the middle..

Question 4

The 5 most common marks in this question were, in descending order of frequency: 6; 7; 8; 9 and 11 equally. This question was expected to challenge candidates, not because of excessive difficulty but because of the somewhat complicated algebra. However any candidate who works carefully and systematically should be able to succeed and it is clear that this happened. It was intended that the way would be eased for candidates who had difficulties in one part to continue on the next by providing the answer at each stage. Parts (a), (b) and (e) were easily handled by most candidates. In (c) the main difficulty noticed was the application of the chain rule. The algebra frequently got in the way of candidates in (d) and (f). A pleasing number put up sound reasoning and understanding for (e) and (f).

Question 5

The 4 most common marks in this question were, in descending order of frequency: 20 and 18 equally; 16 and 14 equally. The underlying theme in this question was that the volume of an unbounded solid might be finite while the surface area may be infinite. Parts (a), (b), (c) and (e) were well done, though some candidates claimed that the integral of $y^{-2}$ is $\ln y^2$. Part (d) was found to be harder, but of course there is some more complicated algebra and arithmetic here. There were some very sensible answers observed in (f). In a couple of places candidates were required to show an understanding of an infinite limit. In part (c) this was well handled but in part (f), while many realised that the volume had a finite limit and the surface area increased, few realised that the surface area increased without limit.

Question 6

The 5 most common marks in this question were, in descending order of frequency: 0; 5; 4; 1 and 3. Most candidates worked through the questions in order and it appears that candidates were out of time or just exhausted by the time they reached this question so did not do it justice. About 40% of those who tackled (a) tried to solve the equation rather than the simpler task of showing by substitution that the given solution really is a solution. Those who did try to solve could rarely manage the partial fractions. Part (b) was the main source of marks in this question, but even then some went about the part in a long-winded way. Part (c) was also popular, but too many reversed the logic and started with $N = {}^{4B}/_{3}$ . Parts (d), (e) and (f) were not well done: even some of the good candidates went off at tangents with rates, etc that went nowhere; quite a few recalculated the relevant times even though they were already given; there were also comprehension problems, such as working from 2B to 3B instead of B to 3B or ${}^{2B}/_{3}$ to 2B instead of ${}^{4B}/_{3}$ to 2B , with the latter in particular giving rise to some really nasty calculations. The mark for (g) was awarded for consistency or even an unsupported guess provided it was correct, and there were certainly some guesses. The standard response in (h) was to set ${}^{dN}/_{dt} = 0$, when it is actually the derivative of this rate which should be 0.

Comments from the Schools:

By and large the comments from the Schools indicated that the examination was at about the right level. In particular the attempt to start questions with relatively easy parts and end with challenging parts was approved. The major criticism was that there were too many small parts of questions. It was also felt that candidates who had studied certain other subjects were at an advantage: this applied to questions 3 (Computer Science) and 4 (Physics). Another point made by a number of Schools was that quite a few questions seemed to be hard at first sight but turned out to be reasonably easy; I make no apology for this as I feel that candidates should not preconceive ideas about the difficulty of a question from a cursory reading of the question, but I do acknowledge that this can lead to some problems where there is a choice of questions. This leads to another comment from a number of Schools: some approved of the choice of 5 questions from 6 and others thought that there should be just 5 questions.

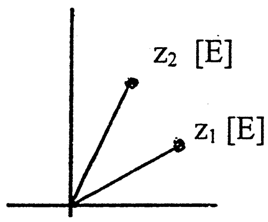
Professor David Gauld, Dept of Mathematics, The University of Auckland.

# NZEST Scholarship Examination 1995
# Mathematics with Calculus
# Solutions and Mark Scheme

Marks are allocated in 3 grades : easy, medium and hard [denoted E,M,H respectively]. E indicates the mark is to be given if the answer is on the right track. M indicates the mark is given provided the answer is almost correct. H means that the mark is reserved for an error-free answer. In some cases (eg 1(b)), some marks are given as E and some as H. This indicates marks are given freely at first, but full marks are only to be given for a completely error-free answer.
Many students will utilise methods of solution different from what is given here. Every effort is made to assess each student fairly at a comparable level to the treatment outlined below.

**Question 1**

(a) (i)




(ii) [E] Reflection in the line $\theta = \pi/4$ (or the line $y = x$)
(or replacing $\theta$ by $\pi/2 - \theta$ .)

(b) [EEM] $i \operatorname{cis}(-\theta) = i\cos(-\theta) + i^2 \sin(-\theta) = i\cos\theta + \sin\theta = \operatorname{sic}\theta$ .

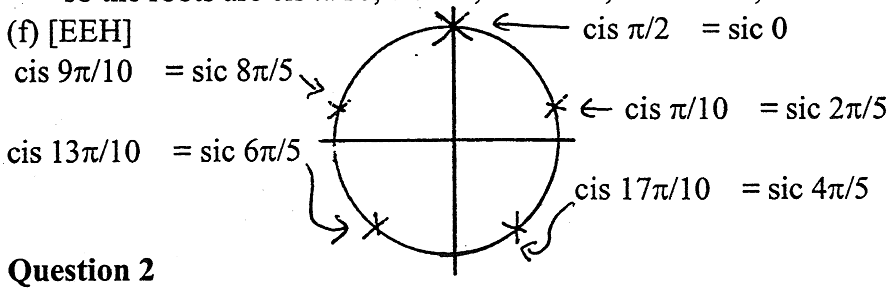
(c) [EEH] $\operatorname{sic}^n \theta = i^n \operatorname{cis}^n(-\theta) = i^n \operatorname{cis}(-n\theta)$.

(d) (i) [EM] $\operatorname{sic}^5 \theta = i^5 \operatorname{cis}(-5\theta)$, so $\operatorname{sic}^5 0 = i^5 \operatorname{cis}(0) = i$ .

(ii) [EH] $\operatorname{sic}^5 2\pi/5 = i \operatorname{cis}(-5.2\pi/5) = i \operatorname{cis}(-2\pi) = i$ .

(e) [EEMH] Write $z = \operatorname{cis}\theta$ . Then $\operatorname{cis}^5 \theta = i$, so $\operatorname{cis}(5\theta) = i = \operatorname{cis}\pi/2$. Hence
$5\theta = 2n\pi + \pi/2$ with $n = 0,1,2,3,4$.
Thus $\theta = 2n\pi/5 + \pi/10$
so the roots are $\operatorname{cis}\pi/10$, $\operatorname{cis}\pi/2$, $\operatorname{cis} 9\pi/10$, $\operatorname{cis} 13\pi/10$, $\operatorname{cis} 17\pi/10$.

(f) [EEH]




**Question 2**

(a) [E] $e = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \ldots$

(b) [E] $s(4) = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} = 2.666\ldots$

(c) [EE] $e = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \ldots + \frac{1}{(n-1)!} + \frac{1}{n!} + \frac{1}{(n+1)!} + \ldots$

$$= s(n) + \frac{1}{n!} + \frac{1}{(n+1)!} + \ldots$$

As each $\frac{1}{m!}$ is positive for m ≥ n, it follows that $\frac{1}{n!}+\frac{1}{(n+1)!}+\ldots > 0$.

So e > s(n), ie s(n) is always too small.

(d) [E] $s^*(4) = s(4) + \frac{1}{3!} = 2.8333\ldots$

(e) (i) [EEH] $\frac{1}{2n!}+\frac{1}{4n!}+\frac{1}{8n!}+\frac{1}{16n!}+\ldots$ is a geometric series

with $a = \frac{1}{2n!}$ and $r = \frac{1}{2}$.

Thus the sum is $\dfrac{\frac{1}{2n!}}{1-\frac{1}{2}} = \frac{1}{n!}$ as required.

(ii) [EMH] From (i), $\frac{1}{(n-1)!} = \frac{1}{2(n-1)!}+\frac{1}{4(n-1)!}+\frac{1}{8(n-1)!}+\frac{1}{16(n-1)!}+\ldots$

When n > 1, n ≥ 2, so 2(n-1)! ≤ n! and $\frac{1}{2(n-1)!} \geq \frac{1}{n!}$.

Also if n ≥ 2 then n+1 > 2, so 4(n-1)! = 2.2(n-1)! < (n+1)n(n-1)! = (n+1)!

So $$\frac{1}{4(n-1)!} > \frac{1}{(n+1)!}.$$

Also if n ≥ 2 then n+2 > 2, so 8(n-1)! = 2.2.2(n-1)! < (n+2)!

So $$\frac{1}{8(n-1)!} > \frac{1}{(n+2)!}.$$

Thus $\frac{1}{(n-1)!} > \frac{1}{n!}+\frac{1}{(n+1)!}+\frac{1}{(n+2)!}+\frac{1}{(n+3)!}+\ldots$

(f) [EEM] $s^*(n) = s(n) + \frac{1}{(n-1)!} > s(n)+\frac{1}{n!}+\frac{1}{(n+1)!}+\frac{1}{(n+2)!}+\frac{1}{(n+3)!}+\ldots = e$

So s*(n) is always too large.

(g) [H] (s(4), s*(4)) = (2.666..., 2.833...)

(h) [EM] Length = s*(n) - s(n) = $\frac{1}{(n-1)!}$.

As $n \to \infty, (n-1)! \to \infty and \frac{1}{(n-1)!} \to 0$.

(i) [EEM] A fraction in the form a/m is also A/m! where A = a(m-1)!.

s(n) and s*(n) are both finite sums of fractions having common denominator (n-1)!.

Then $s(n) = \frac{A_n}{(n-1)!}$ and $s^*(n) = \frac{A_n+1}{(n-1)!}$

If e were a fraction then it could be written e = B/(n-1)! for some integers B and n-1.

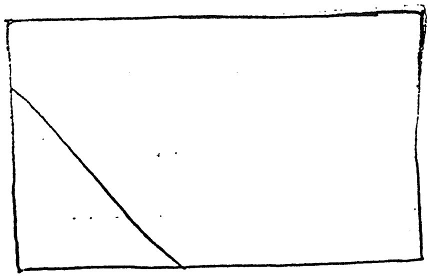
Since s(n) < e < s*(n) it follows that

$\frac{A_n}{(n-1)!} < \frac{B}{(n-1)!} < \frac{A_n+1}{(n-1)!}$ and thus $A_n < B < A_n + 1$, which is impossible

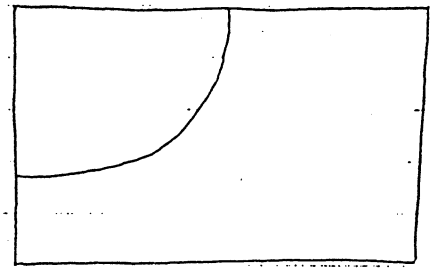
as $A_n$ and $A_n + 1$ are consecutive integers. Thus e cannot be a fraction.

**Question 3**

(a) [E]



(ii) [E]



(b) (i) [H] $x = 320$

(ii) [H] $X = -10$

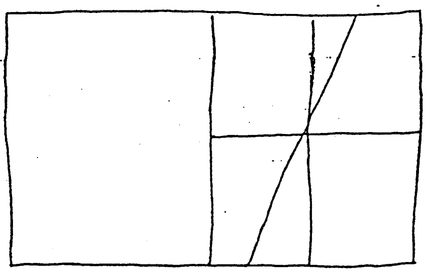
(iii) [HH] $y = 240$, $Y = 0$

(c) (i) [EM] $X = \dfrac{20}{320}(x - 480) = \dfrac{x}{16} - 30$.

(ii) [EM] $Y = \dfrac{20}{480}(240 - y) = 10 - \dfrac{y}{24}$.

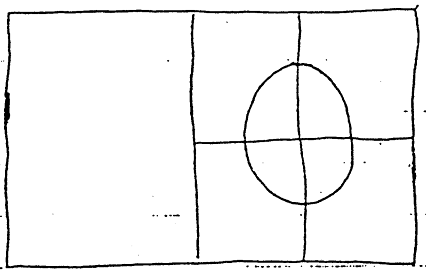
(iii) [HH] For a point to have the same coordinates in both systems, we need $X = x$ and $Y = y$. But $X = x$ requires $x = \dfrac{x}{16} - 30$, which has the solution $x = -32$, which is off the screen.

(d) (i) [M]



(ii) [MH] $Y = 2X + 1$ implies $10 - \dfrac{y}{24} = 2(\dfrac{x}{16} - 30) + 1$, that is $y = -3x + 1656$.

(e) (i) [EE]

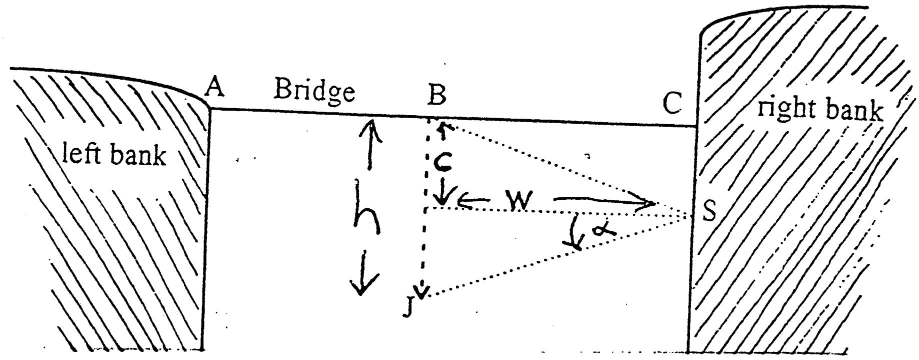


(ii) [H] $(\dfrac{x}{16} - 30)^2 + (10 - \dfrac{y}{24})^2 = 25$, so $\dfrac{(x-480)^2}{80^2} + \dfrac{(y-240)^2}{120^2} = 1.$

(iii) [HH] $\dfrac{120}{80}.10 = 15 and \dfrac{120}{80}.(-10) = -15$, so the minimum and maximum values of the Y-axis should be -15 and 15.

**Question 4**

(a) [HHH]




(b) [EEM] $\tan\alpha = \dfrac{h-c}{w} = \dfrac{\frac{gt^2}{2}-c}{w} = \dfrac{gt^2-2c}{2w}$, *so* $\alpha = \tan^{-1}\dfrac{gt^2-2c}{2w}$.

(c) [EEMM] Let $x = \dfrac{gt^2-2c}{2w}$. Then $\alpha = \tan^{-1}x$, and

$$\frac{d\alpha}{dt} = \frac{d\alpha}{dx}.\frac{dx}{dt}$$

$$= \frac{1}{1+x^2}.\frac{2gt}{2w}$$

$$= \frac{1}{1+\left(\frac{gt^2-2c}{2w}\right)^2}.\frac{gt}{w}$$

$$= \frac{4wgt}{4w^2+\left(gt^2-2c\right)^2}.$$

(d) [EEEE] $$\frac{d^2\alpha}{dt^2} = 4wg\left[\frac{\left[4w^2+\left(gt^2-2c\right)^2\right].1 - t.2\left(gt^2-2c\right).2gt}{[4w^2+(gt^2-2c)^2]^2}\right].$$

Then $\dfrac{d^2\alpha}{dt^2} = 0 \text{ if } 4w^2+\left(gt^2-2c\right)^2 = 4gt^2\left(gt^2-2c\right)$.

ie $4w^2 + g^2t^4 - 4gct^2 + 4c^2 - 4g^2t^4 + 8gct^2 = 0$

ie $3g^2t^4 - 4gct^2 - (4w^2 + 4c^2) = 0$

So $t^2 = \dfrac{2gc \pm \sqrt{4g^2c^2 + 12w^2g^2 + 12g^2c^2}}{3g^2}$

$= \dfrac{2gc \pm 2g\sqrt{3w^2+4c^2}}{3g^2}$

$= \dfrac{2c \pm 2\sqrt{3w^2+4c^2}}{3g}$.

As $t^2 \geq 0$, we must take the positive root.

So $t^2 = \dfrac{2c + 2\sqrt{3w^2+4c^2}}{3g}$.

Thus $\tan\alpha = \dfrac{\frac{2c+2\sqrt{3w^2+4c^2}}{3} - 2c}{2w} = \dfrac{\sqrt{3w^2+4c^2}-2c}{3w}$, so

$\alpha = \tan^{-1}\dfrac{\sqrt{3w^2+4c^2}-2c}{3w}$.

(e) [MM] If the spectator is at the level of the bridge, then $c = 0$ and the maximum is

$\alpha = \tan^{-1}\dfrac{\sqrt{3w^2}}{3w} = \tan^{-1}\dfrac{1}{\sqrt{3}} = \pi/6$.

(f) [EM] For $\alpha = \pi/4$ at the maximum we need $\tan\alpha = 1$, so

$$\frac{\sqrt{3w^2+4c^2}-2c}{3w} = 1$$

$$\begin{aligned}
\sqrt{3w^2+4c^2} &= 3w + 2c \\
3w^2+4c^2 &= 9w^2 + 12\,wc + 4c^2\ . \\
12\,c &= -6\,w \\
c &= -w/2
\end{aligned}$$

Thus the spectator should be $w/2$ metres above the bridge level.

(g) [MM] The calculation of the maximum rate of change of $\dfrac{d\alpha}{dt}$ reveals that it occurs at an angle $\alpha$ which is independent of g. Thus changing the value of g will not change the answer in (d). Hence there will no change in (e) or (f) either.

**Question 5**

(a) [MMMH] Volume $= \pi\int_1^h y^2 dx$

$$\begin{aligned}
&= \pi\int_1^h \frac{1}{x^2}dx \\
&= \pi\left[\frac{-1}{x}\right]_1^h \\
&= \pi\,(1 - 1/h)\ \text{cubic metres.}
\end{aligned}$$

(b) [MH] For h = 329, volume $= \pi\,(1 - 1/329) < \pi < 3.2$ . So they have enough.

(c) [MM] For any h, volume $= \pi\,(1 - 1/h) < \pi < 3.2$ . So they have enough.

(d) [EEEMM] Volume of paint $= \pi\int_1^h\left[\left(\frac{1}{x}+\frac{1}{1000}\right)^2 - \frac{1}{x^2}\right]dx$

$$\begin{aligned}
&= \pi\int_1^h\left[\frac{1}{500x}+\frac{1}{1000^2}\right]dx \\
&= \pi\left[\frac{1}{500}\ln x+\frac{x}{1000^2}\right]_1^h \\
&= \pi\left[\frac{1}{500}\ln h+\frac{h}{1000^2}-\frac{1}{1000^2}\right]\ \text{m}^3
\end{aligned}$$

For h = 329, this comes to $\pi\left[\frac{1}{500}\ln 329+\frac{328}{1000^2}\right]$ m$^3$.

(e) [MM] Surface area $= 1000\pi\left[\frac{1}{500}\ln 329+\frac{328}{1000^2}\right] = \pi\left[2\ln 329+\frac{328}{1000}\right]$ m$^2$

(f) [EEEMM] For h = 329, the volume of the tower excluding paint is about $\pi$ cubic metres, while the surface area is greater than $10\pi$ square metres. Depending on the constants of proportionality, one may expect the tower to be blown down in a strong wind, rather than to collapse under its own weight. For larger h, the volume remains bounded by $\pi$ (refer (c)), while the surface area can be shown by working similar to (e) to increase without bound. Consequently, for larger values of h the likehood of it being blown down becomes increasingly greater than that of it falling down under gravity.

**Question 6**

(a) [EEE] If $N = \dfrac{4Be^{4kBt}}{3+e^{4kBt}}$ then

$$\frac{dN}{dt} = 4B\frac{(3+e^{4kBt})4kBe^{4kBt} - e^{4kBt}.4kB.e^{4kBt}}{\left(3+e^{4kBt}\right)^2}$$

$$= \frac{16kB^2e^{4kBt}\left(3+e^{4kBt} - e^{4kBt}\right)}{\left(3+e^{4kBt}\right)^2}$$

$$= \frac{12kBN}{3+e^{4kBt}}$$

$= kN(4B-N)$ , as $4B-N = \dfrac{4B}{3+e^{4kBt}}\left(3+e^{4kBt} - e^{4kBt}\right) = \dfrac{12B}{3+e^{4kBt}}$.

When $t = 0$, $\dfrac{4Be^{4kBt}}{3+e^{4kBt}} = \dfrac{4B}{3+1} = B$ as required.

(b) [EM] $N = 2B$ if $2B = \dfrac{4Be^{4kBt}}{3+e^{4kBt}}$ , ie $3 + e^{4kBt} = 2e^{4kBt}$.

So $e^{4kBt} = 3$

and thus $t = \dfrac{1}{4kB}\ln 3$ .

(c) [EM] If $t = \dfrac{1}{4kB}\ln\left(\dfrac{3}{2}\right)$ then $e^{4kBt} = \dfrac{3}{2}$ .

So $N = \dfrac{4B.\frac{3}{2}}{3+\frac{3}{2}} = \dfrac{4}{3}B$.

(d) [EEM] If 2B cells are harvested from 3B, then B are left, and it will take $\dfrac{1}{4kB}\ln 9$ hours to grow back to 3B cells again.

Thus the average harvest is now $\dfrac{2B}{\frac{1}{4kB}\ln 9}$ cells per hour, that is

$\dfrac{8kB^2}{\ln 9} = \dfrac{4kB^2}{\ln 3}$ cells per hour.

(e) [EMM] Time to build up from $\frac{4}{3}B = 2B - \frac{2}{3}B$ to 2B is $\frac{1}{4kB}\ln 3 - \frac{1}{4kB}\ln\frac{3}{2} = \frac{1}{4kB}\ln 2$ hours.

The average harvest is now $\dfrac{\frac{2B}{3}}{\frac{1}{4kB}\ln 2} = \dfrac{8kB^2}{3\ln 2}$ cells per hour.

(f) [MMM] The average harvest this time is $\frac{dN}{dt}$ when $N = 2B$. This gives

$$k.2B(4B\text{-}2B) = 4kB^2 \text{ cells per hour.}$$

(g) [E] (f) is best.

(h) [EMH] When $\frac{dN}{dt}$ attains its maximum, $\frac{d^2N}{dt^2} = 0$.

Now $\frac{dN}{dt} = kN(4B\text{-}N)$.

So $\frac{d^2N}{dt^2} = k(4B - N - N).\ \frac{dN}{dt} = 2k(2B\text{-}N).\ \frac{dN}{dt}$.

Thus $\frac{d^2N}{dt^2} = 0$ if $N = 2B$.

Using the "first derivative test" we note that if $N < 2B$ $\frac{d^2N}{dt^2} > 0$,

and if $N > 2B$ $\frac{d^2N}{dt^2} < 0$. Thus $N = 2B$ gives the maximum value of $\frac{dN}{dt}$.

Consequently, any other harvesting technique will involve harvesting where $\frac{dN}{dt}$ is less than this maximum.

# NZEST

NEW ZEALAND EDUCATION & SCHOLARSHIP TRUST

*ACADEMIC ACHIEVEMENT BY CHOICE*

## NZEST Grading System

Using the 1995 subject marks, tabled below, you will see that the NZEST Grading System provides parity between subjects: ie, candidates who sat Chemistry ,which had a low mean, did not suffer as the lowest Grade A+ mark of 60 receives 9 scholarship points compared with the lowest Grade A+ mark of 90 in Maths with Calculus which also receives 9 scholarship points.

| Grade | Scholarship Points | Grade | Scholarship Points | Grade | Scholarship Points |
|---|---|---|---|---|---|
| A+ | 9 | B+ | 6 | C+ | 3 |
| A | 8 | B | 5 | C | 2 |
| A- | 7 | B- | 4 | C- | 1 |
| | | | | D | 0 |

| 1995: Subject | No sitting | Mean mark | Top mark | Lowest mark in each grade A+ | A | A- | B+ | B | B- | C+ | C | C- |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Accounting | 204 | 42 | 80 | 69 | 66 | 61 | 59 | 53 | 49 | 45 | 41 | 38 |
| Biology | 416 | 56 | 83 | 75 | 72 | 68 | 64 | 60 | 56 | 52 | 49 | 46 |
| Chemistry | 519 | 36 | 86 | 60 | 57 | 53 | 49 | 45 | 41 | 37 | 33 | 30 |
| Classical Studies | 125 | 61 | 91 | 85 | 81 | 76 | 71 | 66 | 60 | 56 | 51 | 47 |
| Economics | 220 | 40 | 67 | 57 | 55 | 52 | 49 | 46 | 43 | 40 | 37 | 35 |
| English | 358 | 54 | 91 | 75 | 71 | 67 | 63 | 59 | 55 | 51 | 47 | 40 |
| French | 103 | 64 | 92 | 83 | 80 | 76 | 72 | 68 | 64 | 60 | 55 | 50 |
| Geography | 110 | 51 | 99 | 87 | 81 | 75 | 69 | 64 | 58 | 52 | 46 | 42 |
| German | 39 | 76 | 95 | 85 | 82 | 78 | 74 | 70 | 66 | 60 | 55 | 50 |
| History | 161 | 56 | 91 | 80 | 75 | 70 | 65 | 61 | 56 | 51 | 46 | 40 |
| History of Art | 60 | 59 | 80 | 75 | 72 | 69 | 66 | 63 | 60 | 58 | 55 | 51 |
| Japanese | 119 | 65 | 95 | 90 | 85 | 80 | 74 | 70 | 64 | 58 | 52 | 46 |
| Latin | 11 | 69 | 85 | 85 | 81 | - | 72 | 69 | 60 | 57 | - | - |
| Maori | 6 | 45 | 72 | - | - | - | 72 | - | 64 | - | 50 | 42 |
| Maths/Calculus | 628 | 53 | 100 | 90 | 83 | 76 | 69 | 62 | 55 | 48 | 41 | 34 |
| Maths/Statistics | 588 | 49 | 95 | 80 | 74 | 68 | 62 | 56 | 50 | 44 | 38 | 32 |
| Physics | 481 | 51 | 99 | 88 | 82 | 75 | 69 | 62 | 55 | 48 | 41 | 32 |

**Note re low Chemistry mean:**

The NZEST Academic Committee would like to advise that it is very concerned about the continuing low mean in Chemistry. A new examination team has been appointed for 1996 and they are quietly confident that the mean can be lifted to a reasonable level without in any way dropping the scholarship standard.