NZEST

NEW ZEALAND
EDUCATION & SCHOLARSHIP TRUST

ACADEMIC ACHIEVEMENT BY CHOICE

NZEST SCHOLARSHIP EXAMINATION

1996 EXAMINER'S REPORT AND SOLUTIONS

MATHEMATICS WITH CALCULUS

NZEST Mathematics with Calculus Examiners' Report, 1996

The first priority of this paper remains the same, that is to give average and above average seventh formers an opportunity to demonstrate their creativity and mental strength in a range of different types of problem solving situations. An effort has been made to retain a general structure and plan similar to that of the last few years. Recent changes in the syllabus have been implemented. However, the assessment of competence across the broad range of syllabus content remains a secondary priority. Once again students were invited to attempt all 6 of the 20-mark questions, with their final mark [as a percentage] being [except in a handful of cases over 100] the actual sum of the marks they earned. We believe this procedure is appropriate giving due consideration to the unexpected and challenging nature of most of the paper.

Exam Statistics
Number sat : 767
Median : 52 %
Upper quartile : 66 %
Lower quartile : 37 %

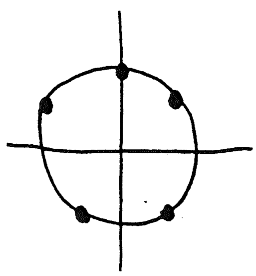
Percentage :	0-	10-	20-	30-	40-	50-	60-	70-	80-	90- 100
Frequency :	8	35	67	106	134	153	113	74	48	29

As you can see the general standard of candidate remains very good despite the continuing increase in the number sitting. Most of the marking panel considered this paper slightly harder than last year's. So it is clear the majority of students can be pleased with their performance. All students achieving over 40% have demonstrated commendable problem-solving abilities in at least some of the questions, and should have benefited from the experience. In the detailed markers' comments below you will notice repetitive comments about weaknesses in certain areas of basic skills.
Once again markers allocated marks in 3 grades :
E indicates an easy mark which will be losely awarded to any approximate or vaguely equivalent working. The intention here is to generously reward students for any sensible attempts at getting started. All candidates should be told clearly about this. If at any stage [especially at the beginning of a question] they are uncertain how to proceed they should be encouraged to jot down any ideas, diagrams, numerical examples, special cases, graphs, etc that come to mind. It is highly likely that such material will earn them some marks, and certain it will not be penalised in any way. Teachers may consider using a similar style of marking for problem-solving sections of school exams.
M indicates a medium mark awarded for a reasonably accurate [but only moderately difficult] step towards the solution.
H are hard marks only awarded for work of considerable ingenuity or high technical class.
In addition, any student demonstrating exceptional originality or viruosity at any stage in any question will have the symbol '+' appended to their score in that question.

Question 1

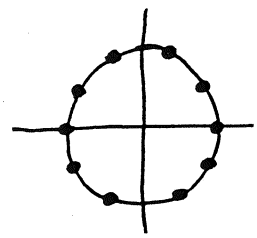
Statistics : median : 6, LQ : 3, UQ : 9, number scoring 20 : 2

Marking scheme :
1(a) [E] As $1 = \omega^n = cis(n\theta) and \varpi = cis(-\theta) then \varpi^n = cis(-n\theta) = \overline{cis(n\theta)} = 1$. (√)
(b) [EH]



(√)(√)

(c) [M]



(√)

(d) [EM] As n is even, n = 2m for some integer m. (√)

Then $(-\omega)^n = (-\omega)^{2m} = (-1)^{2m}\omega^{2m} = \omega^n = 1.$ (√)

(e) [EEM] Let the roots be $\omega_1,\ldots,\omega_{2m}$, (√) where n = 2m.

They may be arranged so that $\omega_{m+j} = -\omega_j$ (√) for each j = 1, ... , m.

Then $\sum_{j=1}^{2m}\omega_j = \sum_{j=1}^{m}\left(\omega_j + \omega_{m+j}\right) = 0.$ (√)

(f)(i)[MH] $f(1) = 1 - 1^n = 0.$ (√)

Thus by the remainder theorem (1-x) is a factor of f(x). (√)

(f)(ii)[M] Multiplying out we get

$(1-x)(1+x+x^2+ \ldots +x^{n-1}) = (1+x+x^2+ \ldots +x^{n-1}) - x(1+x+x^2+ \ldots +x^{n-1}) = 1-x^n = f(x).$ (√)

(g) (i) [M] $f(z) = 1 - \left(cis\left(\frac{2\pi}{n}\right)\right)^n = 1 - cis(2\pi) = 0.$ (√)

(g)(ii) [M] From (f) either 1-z = 0, or $1+z+z^2+ \ldots +z^{n-1} = 0.$

We cannot have 1 - z = 0 as $z = cis\left(\frac{2\pi}{n}\right) \neq 1.$

Thus $1+z+z^2+ \ldots +z^{n-1} = 0.$ (√)

(h) [EM] Let z be as in (g). Then $\{z^k: k = 0, \ldots, n-1\}$ are all nth roots of 1, and they are all different. So they make up all the nth roots of 1. (√) Thus from (g)(ii) the sum of all nth roots is 0. (√)

(i) [EE] Let $z = cis\left(\frac{\pi}{5}\right)$. Then $z^{10} = 1$. So z is a 10th root of 1.

Thus $1+z+z^2+ \ldots +z^9 = 0.$ (√) .....[1]

Now $\{1, z^2, z^4, z^6, z^8\}$ are the 5th roots of 1. So $1 + z^2 + z^4 + z^6 + z^8 = 0.$ .....[2]

Subtracting [2] from [1] yields the result. (√)

(j) [EM] Let c be any complex number. If c = 0 then all the n-th roots of c are 0, and the sum is 0.

If c is not 0 then c = r cis θ for some r>0 and some θ. Let $z = cis\left(\frac{2\pi}{n}\right)$. as in (g).

Let $d = \sqrt[n]{r}\,cis\left(\frac{\theta}{n}\right)$ (√) where $\sqrt[n]{r}$ denotes the positive real nth root of r.

Then the distinct nth roots of c are $d, dz, dz^2, \ldots, dz^{n-1}$.

Note $d + dz + dz^2 + \ldots + dz^{n-1} = d(1+ z + z^2 + \ldots + z^{n-1}) = 0.$ (√)

Markers' comments

This was a difficult question. 1(a) was a bad start to the paper for most. Many could find ϖ and ω^n without reaching a conclusion. Some elegant solutions were based on $\omega.\varpi = 1$ or $\arg\left(\omega^n\right) = 2k\pi$.

(b) and (c) were generally successful and marked generously.

(d) We had difficulty justifying 2 marks as a 2 line solution could get full marks. The crux appears to be stating that the square of a negative is positive. Some gave nice geometric arguments. Unfortunately a lot of students could not distinguish between $-\omega$ and ϖ.

(e) 3 marks is a bit generous. Some students tried to prove the result by appealing to an example.

(f)(i) Some showed a poor understanding of the term "factor".

(ii) Some applied the formula for the sum of a G.P. without due explanation.

(g) Was a lot of work for just 1 mark. Many failed to nail it.

(h) Proved to be very difficult.

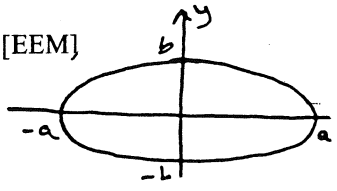
(i) Although hard, there were a number of interesting valid answers given.

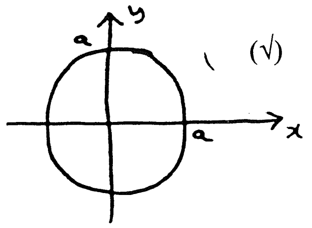
(j) Very few reached this part. It is suggested the desire to set a continuous question may have been stretched too far.

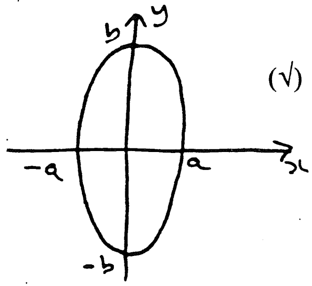
Question 2

Statistics : median : 9, LQ : 5, UQ : 13, number scoring 20 : 2

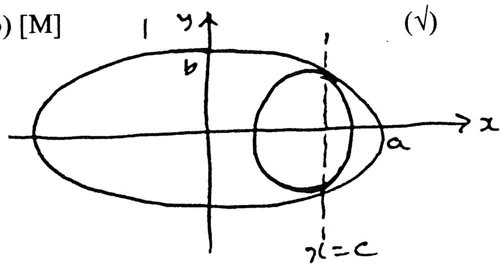
Marking scheme :

(a) [EEM] 

(√) 

(√) 

(√)

(b) [M] 

(√)

(c) [M] Because the ellipse is symmetric about the x-axis (terms involving y in the defining equation have even powers), it follows that the circle must be symmetric about the x-axis. Thus the centre must be at (d, 0) for some d. If the radius is R then the circle has equation $(x-d)^2 + y^2 = R^2$.
$d > 0$ as $c > o$ is given. Clearly $R > 0$. (√)

(d) (i) [EM] Differentiation yields :

$$\frac{2x}{a^2} + \frac{2yy'}{b^2} = 0, \quad \text{so} \quad y' = -\frac{b^2}{a^2}\frac{x}{y}. \qquad (\surd)$$

When x = c, $y = \pm b\sqrt{1 - \frac{c^2}{a^2}}$. So the gradients of the tangents are :

$$\mp\frac{b^2}{a^2}\frac{c}{b\sqrt{1-\frac{c^2}{a^2}}} = \mp\frac{bc}{a\sqrt{a^2-c^2}} \quad . \qquad (\surd)$$

(d) (ii) [EM] Differentiation yields :

$2(x-d) + 2yy' = 0$. So $y' = -\dfrac{x-d}{y}$. (√)

When x = c, $y = \pm\sqrt{R^2 - (c-d)^2}$.

So the gradients of the tangents are $\mp\dfrac{c-d}{\sqrt{R^2-(c-d)^2}}$. (√)

(d) (iii) [EMH] The circle and the ellipse have common tangents. So the gradients must be the same.

$$\frac{bc}{a\sqrt{a^2-c^2}} = \frac{c-d}{\sqrt{R^2-(c-d)^2}} \qquad (\surd) \qquad \text{................[1]}$$

The y-coordinates when x=c must be the same.

$$b\sqrt{1-\frac{c^2}{a^2}} = \sqrt{R^2-(c-d)^2} \ . \qquad (\surd)$$

Thus $\frac{b}{a}\sqrt{a^2-c^2} = \sqrt{R^2-(c-d)^2}$ ....[2]

Substituting [2] into [1] yields

$$\frac{bc}{a\sqrt{a^2-c^2}} = \frac{a(c-d)}{b\sqrt{a^2-c^2}}\ .$$

So $b^2c = a^2(c-d)$, and hence $a^2d = c(a^2 - b^2)$, and $d = c(1 - b^2/a^2)$. (√)

(d) (iv) [MH] Substituting the solution for (d) in [2] gives

$$R^2 = (c-d)^2 + \frac{b^2}{a^2}\left(a^2-c^2\right) \qquad (\surd)$$

$$= \frac{c^2b^4}{a^4} + b^2 - \frac{b^2c^2}{a^2}$$

$$= \frac{b^2}{a^4}\left(b^2c^2 + a^4 - a^2c^2\right)$$

So $R = \frac{b}{a^2}\sqrt{\left(b^2c^2 + a^4 - a^2c^2\right)}$. (√)

(e) (i) [H] Setting c = 0 in the formula for R gives $R = \frac{b}{a^2}\sqrt{a^4} = b$. (√)

(e) (ii) [MH] As $c \to a$, $R \to \frac{b}{a^2}\sqrt{\left(b^2a^2 + a^4 - a^2a^2\right)}(\surd) = \frac{b^2}{a}$. (√)

(f) [MHH] As $k = \frac{\pi R^2}{\pi ab} = \frac{b^2\left[a^4 - \left(a^2-b^2\right)c^2\right]}{a^4.ab} = \frac{b}{a^5}\left[a^4 - \left(a^2-b^2\right)c^2\right]$. (√)

It follows that as c increases, k decreases. Thus k is at least the value when $c \to a$, and at the most the value when c = 0. (√)

Hence $\frac{ba^2b^2}{a^5} \le k \le \frac{ba^4}{a^5}$. So $\frac{b^3}{a^3} \le k \le \frac{b}{a}$. (√)

Markers comments :

Almost every student could draw the 3 graphs in (a).
(b) was also easy, although some felt the centre of the circle needed to be at (0, 0).
(c) was done poorly, usually because they failed to make any reference to the physical nature of the problem.
(d) Many could not do implicit differentiation. Even more were unable to find the gradient at x = c.
In part (iii) some could see they needed to equate the tangents, but found the algebraic manipulation too hard.
(e) Those still on track did well here. Providing the answer was a good idea.
(f) Very few could manage this. Those that did gave some splendid answers.

Question 3

Statistics : median : 10, LQ : 7, UQ : 14, number scoring 20 : 3

Marking scheme :

(a) [EM] Reading the row with x = 1.3 and column 0 we get ln 1.3 = 0.26236 (√)
ln 13 = ln(10 x 1.3) = ln 10 + ln 1.3 = 2.30259 + 0.26236 = 2.56495 (√)

(b) (i) [E] $1.3 \times 1.2 = 1.56$
$\ln 1.56 = 0.44469$
$\ln 1.3 + \ln 1.2 = 0.26236 + 0.18232 = 0.44468$ (√)

(b) (ii) [EM] $\ln(2.43 / 2.13) = \ln 2.43 - \ln 2.13 = 0.88789 - 0.75612 = 0.13177$ (√)

From the table $\ln 1.14 = 0.13103$ which is the closest we can get to to 0.13177.
So $2.43 / 2.13 = 1.14$ approximately. (√)

(c) [EH] $\log_{10} 1.6 = [\ln 1.6]/[\ln 10]$ (√) $= 0.47 / 2.30 = 0.204$ (√)

(d) (i) [E] Calculus applications are simpler ... (√)

(d) (ii) [E] Arithmetic calculations are simpler .... (√)

(e) (i) [EM] For $A^x = e^{kx}$ we must have $A = e^k$.
So $\ln A = \ln e^k = k$
Thus $A^x = e^{(\ln A)x}$. (√)

If $f(x) = A^x = e^{kx}$ where $k = \ln A$,
then $f'(x) = ke^{kx} = (\ln A)A^x$. (√)

(e) (ii) [EEMM] Let m_1 be the slope of $y = e^x$ at (0, 1), and m_2 the slope of $y = 10^x$ at (0, 1).
Then when $y = e^x$, $y' = e^x$. So $m_1 = 1$. (√)

Also when $y = 10^x$, $y' = (\ln 10)10^x$. So $m_2 = \ln 10$. (√)

Then $\dfrac{-m_1 + m_2}{1 + m_1 m_2} = \dfrac{\ln(10) - 1}{\ln(10) + 1}$. (√)

So the angle is $\tan^{-1}\left(\dfrac{\ln(10) - 1}{\ln(10) + 1}\right)$. (√)

(f) [EEEMH]

Area between curves for $-k \le x \le 0$: $\displaystyle\int_{-k}^{0} \left(e^x - 10^x\right) dx$ (√) $= \left[e^x - \dfrac{10^x}{\ln 10} \right]_{-k}^{0}$ (√)

$= 1 - 1/\ln 10 - e^{-k} + 10^{-k} / \ln 10$

$= \dfrac{\ln(10) - 1}{\ln 10} - \left(e^{-k} - \dfrac{10^{-k}}{\ln 10} \right)$. (√)

For any positive k, $10^{-k} < e^{-k}$. So $\left(e^{-k} - \dfrac{10^{-k}}{\ln 10} \right) > 0$. (√)

Thus the area is always less than $\dfrac{\ln(10) - 1}{\ln 10}$. (√)

Markers comments :

(a) Very well done. There were various interpretations of 'explain'. Some thought this meant 'write an essay'. Our interpretation is that an explanation does not have to be all words, but may consist almost entirely of appropriate mathematical calculations.

(b) & (c) were both well done.
(d) Wide, varied and often humourous answers were marked compassionately. Here are a few of the best :
Why do mathematicians prefer natural log tables ?
"They enjoyed the complexities of calculating using the natural log values as they needed something to do to fill in the day. Golf was not a common game in those times."
"Because their friend who invented it was a mathematician and mathematicians stick together."
Why do other people prefer common log tables ?
"Normal people have always considered numbers like 10 to be normal; they have 10 fingers."
(e) (i) It was often hard to tell if a candidate was working back from the answer.
(e) (ii) Was well done, though some forgot $x = 0$.
(f) Very few got this completely correct, with 2 a common score.

Question 4

Statistics : median : 13, LQ : 8, UQ : 16, number scoring 20 : 57

Marking scheme :

(a) [EEMM] $\dfrac{dP}{dt} = \dfrac{P}{200}$. So $\int \dfrac{dP}{P} = \int \dfrac{dt}{200}$. (√)

Thus $\ln P = t/200 + C$. (√)

Hence $P = P_0 e^{t/200}$. (√)

When $t = 0$, $P = 1$. So $P_0 = 1$. Thus $P = e^{t/200}$. (√)

(b) [EM] When $t = 100$, $P = e^{1/2}$ (√) $= 1.648$ (approx).
Thus there will be about 1648 possums. (√)

(c) [EEM] When $P < 3$, $dL/dt > 0$. So the number of leaves increases. (√)
When $P > 3$, $dL/dt < 0$. So the number of leaves decreases. (√)
The number of leaves will reach their maximum when $P = 3$.
So $e^{t/200} = 3$, giving $t = 200 \ln 3 = 220$ approximately. (√)
Thus the maximum number of leaves will be attained after 220 days.

(d) (i) [EM] When $L = 3t - 200\, e^{t/200} + C$ we have $\dfrac{dL}{dt} = 3 - \dfrac{200}{200} e^{t/200}$ (√) $= 3 - P$. (√)

(ii) [EM] When $t = 0$, $L = 1000$. So $1000 = 0 - 200\, e^{0} + C$. (√)
Thus $C = 1000 + 200 = 1200$.
Hence $L = 3t - 200\, e^{t/200} + 1200$. (√)

(e) [EMH] When $t = 220$ and $P = 3$ (√) we have $L = 660 - 600 + 1200 = 1260$. (√)
Thus the maximum number of leaves will be 1260 million. (√)

(f) [EEMH] On 1/3/98, $t = 19 + 31 + 31 + 28 + 365 = 474$.
On 31/3/98, $t = 474 + 30 = 504$. (√)
When $t = 474$, $L = 1422 - 200\, e^{2.37} + 1200 = 482$ approximately . (√)
When $t = 504$, $L = 1512 - 200\, e^{2.52} + 1200 = 226$ approximately. (√)
Thus collapse will take place during March 1998 as the number of leaves will drop from 482 million to 226 million during this month. (√)

Markers comments :

Many found the distinction between thousands and millions confusing.
Some indulged in inappropriate rounding. Some could not understand that p is a function of t.
Overall, most found this rather routine question very easy. Significantly, the interesting and elaborate story does not seem to have intimidated anyone.

Question 5

Statistics : median : 7, LQ : 5, UQ : 11, number scoring 20 : 5

Marking scheme :

(a) [EEM] $f(x) = (2x)^2 = 4x^2$.

So $\dfrac{f(x+h)-f(x)}{h} = \dfrac{4(x+h)^2 - 4x^2}{h}$ (√)

$= [8xh + 4h^2]/h = 8x + 4h$. (√)

Hence $f'(x) = \lim\limits_{h\to 0}(8x+4h) = 8x$. (√)

(b) (i) [H] $(u-v)(u^2 + uv + v^2) = u^3 + u^2v + uv^2 - u^2v - uv^2 - v^3 = u^3 - v^3$. (√)

(b) (ii) [EEM] $\dfrac{g(x+h)-g(x)}{h} = \dfrac{\sqrt[3]{x+h} - \sqrt[3]{x}}{h} = \dfrac{u-v}{h}$ (√)

$= \dfrac{u^3 - v^3}{h\left(u^2 + uv + v^2\right)}$ (√) $= \dfrac{x+h-x}{h\left(u^2+uv+v^2\right)} = = \dfrac{1}{u^2+uv+v^2}$.

Hence $g'(x) = \lim\limits_{h\to 0}\dfrac{1}{u^2+uv+v^2} = \dfrac{1}{3\sqrt[3]{x^2}}$. (√)

(b) (iii) [EMH] $\dfrac{g(0+h)-g(0)}{h} = \dfrac{\sqrt[3]{h} - 0}{h}$ (√) $= h^{-2/3}$. (√)

However, $\lim\limits_{h\to 0}\left(h^{-2/3}\right)$ does not exist. So g is not differentiable at 0. (√)

(c) (i) [EEMH] At $x = 1$, $3x^{2/3} = 3$, and $k - (x-m)^2 = k - (1-m)^2$.
Thus, for continuity at 1 we need $k - (1-m)^2 = 3$. (√) ....(1)
At $x = -1$, $3x^{2/3} = 3$, and $k - (x+m)^2 = k - (-1+m)^2$.
So continuity is guaranteed at -1 too.
For smoothness at $x = 1$, we need the derivatives of $3x^{2/3}$ and $k - (x-m)^2$ to be the same.
So $-2(1 - m) = 2$ (√) ....(2)
For smoothness at $x = -1$ we have similar requirements which are equivalent to (2).
Continuity for all other x and smoothness at all other x except 0 follows from continuity and smoothness of the pieces.
Thus we need precisely (1) and (2) to be satisfied.
From (2), $m = 2$. (√)
Hence from (1), $k = 4$. (√)
(c) (ii) [EM] Since h is an even function, if we find x-intercepts for $x >= 0$, the rest is easy.
For $x > 1$, $h(x) = 0$ implies $4 - (x-2)^2 = 0$. So $x = 4$. (√)
For $0 <= x <= 1$, $h(x) = 0$ implies $3x^{2/3} = 0$. So $x = 0$.
Thus the intercepts are $x = -4$, $x = 0$ and $x = 4$. (√)

(c) (iii) [EMMH] Area = $\displaystyle\int_0^1 3x^{2/3}dx + \int_1^4 \left(4-(x-2)^2\right)dx$ (√)

$$= \left[\frac{9}{5}x^{5/3}\right]_0^1 + \left[2x^2 - \frac{x^3}{3}\right]_1^4 \qquad (\surd)$$

$$= 9/5 \;+\; 32 \;-\; 64/3 \;-\; 2 \;+\; 1/3 \qquad (\surd)$$

$$= 54/5 \qquad (\surd)$$

Markers comments :

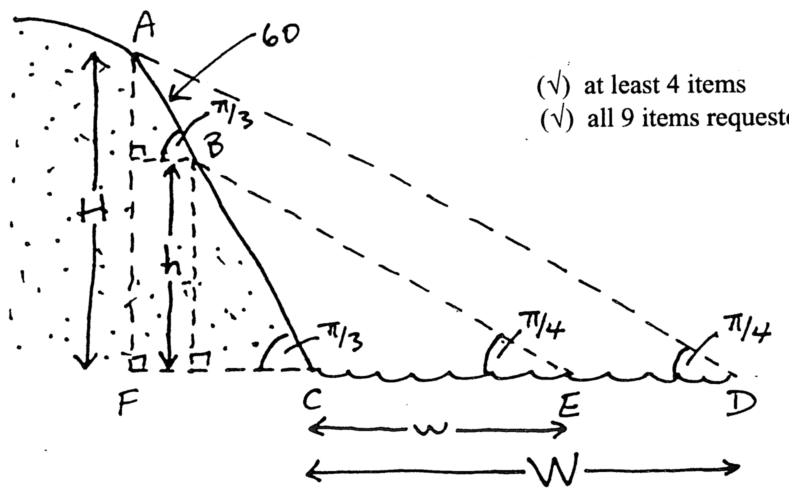
Most made a good attempt at (a) and had clearly been taught the first principles method. Unfortunately, marks were lost for foolish placement of =, and other nonsensical or confused statements involving limits. Poor algebra was too common. Those who knew the answer [by differentiating] and tried to 'fudge' their answer were very obvious.
(b)(i) was done very well.
(b)(ii) Here only the first mark was easy. Only the very best got all 3 marks.
(b)(iii) There were easy marks here as full marks were awarded for simple common-sense.
(c)(i) Many correctly equated function values, but very few considered equating derivative values.
The concept of "smoothness" appeared foreign to most. It was common to see the assumption that "continuous" implies "differentiable".
(c)(ii) was answered well.
(c) (iii) Too many had given up by this stage.

Question 6

Statistics : median : 8, LQ : 4, UQ : 12, number scoring 20 : 2

Marking scheme

(a) [EM]



(√) at least 4 items
(√) all 9 items requested

(b) (i) [EM] tan $\pi/3$ = AF/FC = H/FC.
So FC = H /√3 . (√)
AF = FD. So FD = H. Thus H = H /√3 + W . (√)

(b) (ii) [M] By similar triangles, h = h /√3 + w . (√)

(c) [EEM] Suppose the waves travel at v metres per second. Then the time taken from D to C will be W/v seconds, and from E to C will be w/v seconds.
Thus W/v = 15, and w/v = 12.
So W/w = 15/12 = 5/4 . (√)

Hence $\dfrac{H\left(1-\dfrac{1}{\sqrt{3}}\right)}{h\left(1-\dfrac{1}{\sqrt{3}}\right)}=\dfrac{5}{4}.$ (√) So H/h = 5/4 . (√)

(d) [EMH] H = h + 60 sin π/3 = h + 30√3 (√)
= 0.8H + 30√3 . (√)
Therefore H = 150√3 . (√)

(e) [EM] v = W/15 (√) = H(1 - 1/√3)/15 = 10 √3 (1 - 1/√3) = 10(√3 - 1).
So the waves are traveling at 10(√3 - 1) metres per second. (√)

(f) [EM] $\tan\alpha=\dfrac{H}{\dfrac{H}{\sqrt{3}}+x}=\dfrac{H\sqrt{3}}{H+\sqrt{3}x}$. (√)

Differentiate both sides (implicitly) with respect to t.

$$\sec^2\alpha.\frac{d\alpha}{dt}=\frac{-3H}{\left(H+\sqrt{3}x\right)^2}.\frac{dx}{dt} .$$

But $\sec^2\alpha = \tan^2\alpha + 1 = \left(\dfrac{H\sqrt{3}}{H+\sqrt{3}x}\right)^2+1=\dfrac{3H^2+\left(H+\sqrt{3}x\right)^2}{\left(H+\sqrt{3}x\right)^2}$.

So $\dfrac{d\alpha}{dt}=\dfrac{-3H}{\left(H+\sqrt{3}x\right)^2}.\dfrac{dx}{dt}.\left(\dfrac{3H^2+\left(H+\sqrt{3}x\right)^2}{\left(H+\sqrt{3}x\right)^2}\right)^{-1}=\dfrac{-3H}{\left(H+\sqrt{3}x\right)^2+3H^2}.\dfrac{dx}{dt}$ (√)

(g) [H] $\dfrac{d\beta}{dt}=\dfrac{-3h}{\left(h+\sqrt{3}x\right)^2+3h^2}.\dfrac{dx}{dt}$ (√)

(h) [EEMH] The angle ASB is α -β . So its maximum will occur when $\dfrac{d}{dt}(\alpha-\beta)=0$.

That is when $\dfrac{d\alpha}{dt}=\dfrac{d\beta}{dt}$. (√)

From (f) and (g) :

$$\frac{-3H}{\left(H+\sqrt{3}x\right)^2+3H^2}.\frac{dx}{dt} = \frac{-3h}{\left(h+\sqrt{3}x\right)^2+3h^2}.\frac{dx}{dt} . \quad (\surd)$$

So $H(h + \sqrt{3}x)^2 + 3Hh^2 = h(H + \sqrt{3}x)^2 + 3H^2h$.
Thus $Hh^2 + 2\sqrt{3}Hhx + 3Hx^2 + 3Hh^2 = H^2h + 2\sqrt{3}Hhx + 3hx^2 + 3hH^2$.
Hence $3(H-h)x^2 = 4H^2h - 4Hh^2 = 4Hh(H-h)$.

So $x^2 = (4Hh)/3$. Thus $x = 2\sqrt{\dfrac{Hh}{3}}$. (√)

Now H = 150√3 , and H/h = 5/4 . So h = 120√3 .
Consequently, x = 120√5 . (√)
Thus the angle ASB reaches its maximum when the surfer is 120√5 metres from the shore.

Markers comments :

(a) Students need to be encouraged to draw large diagrams.
(b) Trig ratios were often incorrect, or the wrong one was selected.
The use of similar triangles was too infrequent.
(c) was well done.
(d) and (e) were done well by those who got this far.
(f) was difficult. Some got tan α , but could not differentiate it.
(g) Those who spotted that a simple substitution of h for H was all that was needed got an easy mark.
(h) was very difficult, with some brave souls plunging determindedly into pages of increasingly complex calculations. Only the most able students coped with this part.
It is recommended that some of the angles mentioned be given on the diagram, as "inclined to the horizontal" was not always understood.
Overall, this was a good question. Most could get some marks and must have felt they were getting into the problem. The last part clearly sifted out the most gifted. There were several places where a fresh start could be made even if earlier parts were incorrect.
Students should be encouraged strongly to present a verbal answer as a conclusion to their calculations to each section of a practical problem such as this. One short sentence is almost always all that is required. In this manner they demonstrate that they can see the point in their work.

Feedback from school evaluations.
These responses are always very helpful to examiners and are considered thoughtfully during the construction of the following exam. So those of you who contribute to this are to be sincerely thanked. This year most of the comments were positive and included several explicit requests to continue with this style of paper. Here are some of the specific comments.
The formulae pages need to be revised. Topics nolonger in the syllabus should be deleted.
One of the questions should be presented as a whole, and not be broken down into little steps.
Question 1 was too hard to start with. It should have been swopped with question 4.
Several felt question 6 required the derivative of arctan, which is not in the syllabus [refer marking scheme above where it is solved without differentiating arctan].
Students were thought to have enjoyed the applications questions best, with a number of favourable comments about the log question .

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