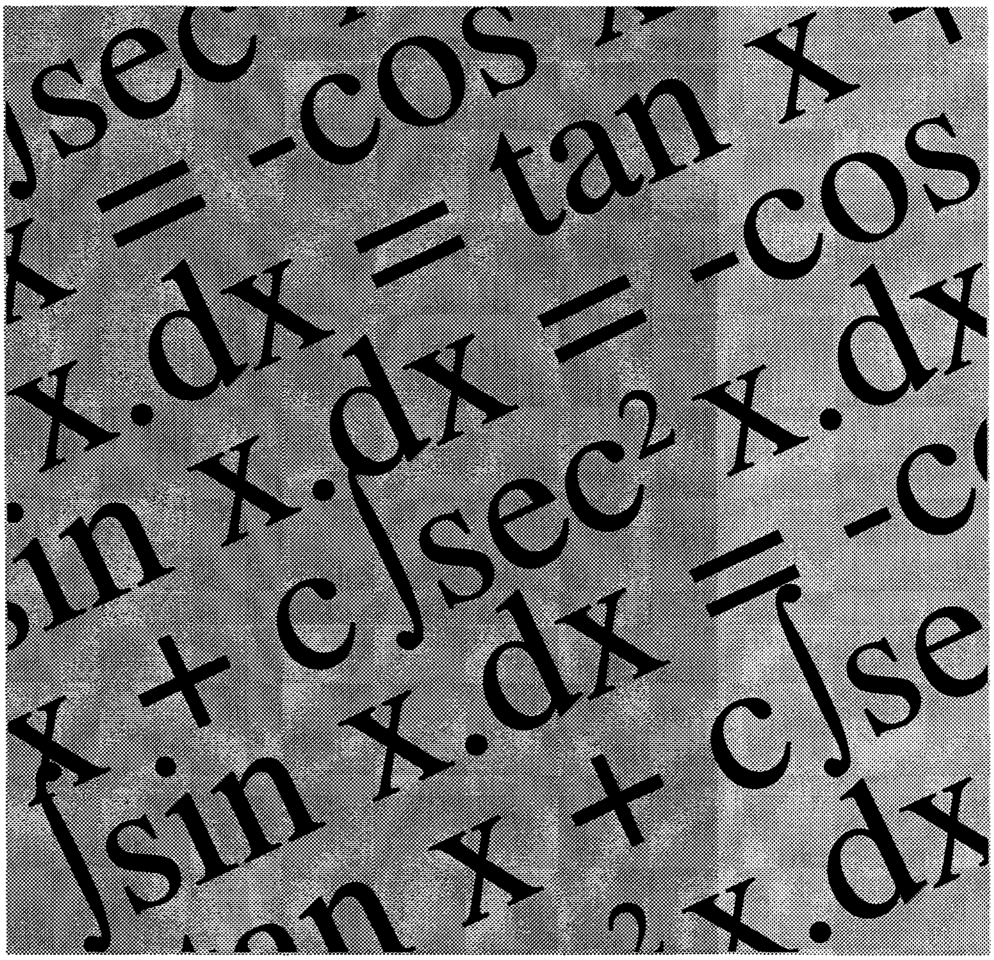
NZEST SCHOLARSHIP EXAMINATION

2001 EXAMINER'S REPORT
AND MARKING GUIDELINES

MATHEMATICS WITH CALCULUS



NZEST

NEW ZEALAND EDUCATION & SCHOLARSHIP TRUST

NZEST EXAMINER'S REPORT

Mathematics with Calculus 2001

Mark Adjusting

Although NZEST does not practise scaling there is a necessity to adjust the calculus marks as there is a total of 120 marks available to the candidates. Five candidates scored more than 100 marks with one candidate getting a perfect score of 120, a most remarkable achievement.

The way in which the marks were adjusted was to use a piecewise function as follows: If a candidate scored 90 or less then their mark was not adjusted. If the candidate scored 91 or more then their new mark was the rounded value of $90+(x-90)\div 3$ where x was the mark they got out of 120.

General Comments

It was agreed by the markers that this was a good paper with enough in it for most candidates to attempt without any part being too easy. The paper challenged the most able students without any impossibly difficult questions. The raw statistics for the adjusted marks are given below.

Statistic	*Value*
Minimum	1
Lower Quartile	26
Median	41
Mean	43
Upper Quartile	57
Maximum	100
Standard Deviation	22

Question 1

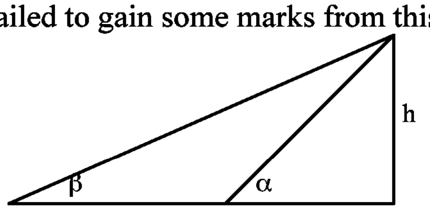
A good question to start the paper. The median obtained was 12 out of 20 with a lowest mark of 0 obtained by 15 of the 649 who attempted this question. The highest mark was 20 which was obtained by 16 candidates.

1(a) The most common error was stopping at cos D = 0.5 which yielded D = $\pi/3$ and not investigating other solutions. Quite a few got $7\div\pi$ as C.

Some wrote A = -3 or B = -2. Few students failed to gain some marks from this part of Q1, most got 4 or more.

(b) There were many variations that worked.

The most common error for the diagram was



However despite having made this initial error the score was usually 3/5.
There were some interesting approaches using Pythagoras or the Sine Rule both of which worked eventually. This question went well with most students getting some marks.

(c) was not done well. Many forgot to square f(x) .

The most common approach by far was to calculate three areas resulting in Trapezium = 3.792, Simpson = 4.009, Integral = 4.

Only the good candidates were able to cope with volume using a numerical method.

There were some interesting approaches in which radians and degrees were mixed up in the same problem.

Question 2

This proved a fairly easy question for the calibre of candidate who entered this exam. The mean mark for this question was 11.5 out of 20 with the full range of marks being scored. 20 was in fact the mode with the marks being distributed fairly evenly between 0 and 20..

2(a)(i) This was straightforward and well done.

2(a)(ii) This was generally well done. Although the correct answer was $|x - b|$ x-b or

b-x were also accepted. The most common error was to give $d_2 = \sqrt{(x - b)^2 + y^2}$

2(a)(iii) This was well done by most candidates however many believe $a^2 = b^2$ implies $a = b$ and were penalised one mark
2(a)(iv) Assuming $a = b$ gave $y^2 = 0$ most candidates took $a = -b$ at this stage with no justification to yield the answer $y^2 = 4ax$.
2(a)(v) This question was done well. A few candidates did not give the coordinates.

2b(i) Common mistakes were to :1) Omit axes 2) Not pass $y^2 = 4ax$ through (0,0)
3) Draw $y = x^2$ 4) Shift $y^2 = -4h(x-c)$ vertically.
2(b)(ii) Very few candidates who did not differentiate implicitly were able to achieve correct solutions. A very common error was to misquote the relationship $mn = -1$.A few candidates used the result 2a(iii).
2(b) (iii)This was done well by those who answered (ii) correctly.

It was felt by those who marked the paper that it would have been better to state at the outset that a is not equal to b. This did cause a lot of problems later on for many candidates.

Question 3

This question was the most poorly done of any in the paper.
The median mark of this question was 2 ,with a lower quartile of 0. The upper quartile was 5 and the mean mark was 3.9. The question was deliberately set with no lead in parts in order that able students might demonstrate their ability to tackle a problem without many hints.

The question was the one which was designed to allow the students maximum flexibility as to how they might tackle it and probably because of this many students made no effective progress. At the same time it proved an effective challenge for the top students and 7 of them gained 20 marks. Only about 10% of candidates gained 10 marks or more.

(a) This part of the question asked "Who is correct ?" Most candidates attempted to find an expression for the area and used calculus to find where the maximum occurred. A small number of able students short circuited the process by finding an x value which gave a greater value than Marsha's. Many wasted time and energy grappling with an intractable second derivative to show a maximum when much simpler arguments would have closed the case.

(b) The introduction of z proved the undoing of many.

A number of candidates mistakenly assumed the maximum volume would occur at the same x value as the maximum area in Q3a.Only a few marks were given for this.

The biggest challenge for most students was setting up the equation for the area and eliminating either x or y to get an expression in one variable.

Question 4

This question was not well done with over 500 of the candidates getting 5 or less. The comments below indicate markers' observations or concerns.

(a) The majority did not use the binomial expansion which should be automatic for good mathematics students. Many used $\sin(3\theta) = \sin(2\theta + \theta)$ etc which if done correctly gained two marks but did not follow instructions.

(b) (i) Using n instead of k caused problems for many in this question. Some did not state k values.
(b)(ii)The majority assumed $z^n = 1$ from part (i). From there they had no chance of further marks. Taking the n^{th} root of both sides gave one solution but in the majority of cases was not continued on to the required solutions. Only the very best candidates were able to finish this.
(c)(i) This was an easy mark for those who attempted it.

(c)(ii)Most did not find that $\frac{1}{(n+1)(n+2)} < \frac{1}{(n+1)(n+1)}$ etc and relate back to part (i).

(c)(iii) There was an easy mark for multiplying by n! which was not often claimed. Very few were able to obtain a correct finish.

Despite all these problems there were two students who got full marks for this challenging question.

Question 5

The median for this question was 7 and the mean mark was 7.4. The lower quartile was 3 and the upper quartile was 11. The mode was 5 and there was a full range of marks from 0 to 20. Sixteen students did not attempt this question.

Q5 was one of the easier questions because candidates were led through it. A significant number of those who attempted the question did not use the leads.

(a)(i)An easy question well handled by most candidates.

(a)(ii)Easily done if handled by inspection rather than substitution.

(a)(iii)Many students did not replace dy with cos φ dφ but rather dφ leading to $\int \frac{\cos\varphi}{\sin\varphi}\,d\varphi$ rather than $\int \frac{\cos^2\varphi}{\sin\varphi}\,d\varphi$. This question was a good test of algebraic and trigonometric skills. Many candidates used a ‘guess and check’ method rather than a more formal mathematical approach.

(b) This was spoilt by careless errors on the part of many even to the extent of transposing x and y.

(c)(i)Most made a connection with (b) and had a consistent even if incorrect answer.
(c)(ii)The instruction *Explain* led to much confusion. Few achieved full marks here.
(c)(iii) Many candidates were able to separate variables and successfully used a(iii) to complete the integration. Many did not evaluate the constant of integration which was sufficient for gaining the last two marks.
c(iv) This was a more difficult calculator exercise. Candidates with a simple, incorrect answer to c(iii) could not score consistency marks here. Students must be trained to instinctively ask “Is this answer sensible ?”. There were some very silly answers which should have been rejected with a bit of reflection.

Question 6

The median mark obtained for this question was 7 out of 20. The lower quartile was 3 and the upper quartile was 10. Three candidates got full marks. The highest mark obtained was 17 out of 20. About fifty students did not attempt this question.

Marker's comments on the individual portions of this question follow:

(a)(i) This was a straightforward question and was well done.

(a)(ii) This was a well done question. The most common error seemed to be giving an example rather than a proper proof.

(b) This part of the question was a simple piece of book work and was well done.

(c)(i) This was straightforward if the cosine rule was used. Some candidates used rather complicated processes involving the distance formula with one using three pages to get the one mark on offer !
(c)(ii)This question was reasonably done with many students appreciating that the product of two tiny quantities will yield a much smaller quantity.

(d)(i) This question, worth one mark, was generously marked and many candidates were able to give a sensible answer.
d(ii) Able students proved quite adept at solving this.
d(iii) There were some good integrations, with many leaving their answer in logarithmic form. A number did not bother to calculate the constant of integration.
d(iv) Knowledge of the spiral was evident among the better answers however there were many who believed that $r = e^{0.75\theta}$ was an exponential curve.
d(v) There were some facetious answers such as 'shoot the police', 'sink the boat and swim', 'pray' etc however so long as the candidates referred to their answer in (iv) they got at least one mark provided the answer was mathematical and not silly.

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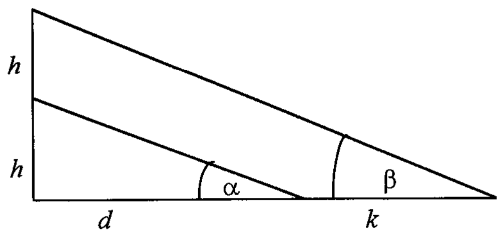
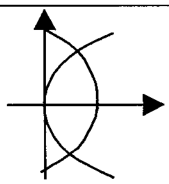
1a	The maximum depth will occur when $Ct+D$ is a multiple of 2π when $\cos(Ct+D)$ will be 1 thus $A+B=5$ For a minimum $Ct+D$ will be an odd multiple of π so $A-B=1$ Solving these equations simultaneously gives $A=3, B=2$. As the period of cosine is 2π $14C=2\pi$ *or* $C=\frac{\pi}{7}$ Initially t = 0 thus substituting we get $3+2\cos D=4$ Thus $\cos D=\frac{1}{2}$ Thus $D=\frac{\pi}{3}$ or $\frac{5\pi}{3}$ However an inspection of the graph shows $D=\frac{5\pi}{3}$.	Various orders for finding ABCD are possible **Total = 7**	1 1 1 1 1 1 1
1b	Let the cliff be of height $2h$  $\frac{h}{d}=\tan\alpha$ and $\frac{2h}{d+k}=\tan\beta$ Thus $\frac{2d}{d+k}=\frac{\tan\beta}{\tan\alpha}$ by dividing one equation by the other $2d\tan\alpha = d\tan\beta + k\tan\beta$ [Cross multiplication] $\therefore$ $k\tan\beta = 2d\tan\alpha - d\tan\beta$ $\therefore k=\frac{d(2\tan\alpha-\tan\beta)}{\tan\beta}$ [Dividing by tan β] hence result	Diagram If no diagram is drawn but problem is correctly solved, give full 5 marks **Total = 5**	1 2 2
1c	Using the trapezium rule $\pi\int_0^{\pi}(2\sin x)^2\,dx = 0.5\pi^2(\sin^2 0$ $+2(\sin^2\frac{\pi}{4}+\sin^2\frac{\pi}{2}+\sin^2\frac{3\pi}{4})+\sin^2\pi) = 2\pi^2$. Using Simpson's rule $\pi\int_0^{\pi}(2\sin x)^2\,dx = \frac{\pi^2}{3}(\sin^2 0 + 4\sin^2\frac{\pi}{4} +$ $2\sin^2\frac{\pi}{2}+4\sin^2\frac{3\pi}{4}+\sin^2\pi) = 2\pi^2$. The volume of revolution is $\pi\int_0^{\pi}(2\sin x)^2\,dx = 4\pi\int_0^{\pi}\sin^2 x\,dx$ $=4\pi\int_0^{\pi}\frac{1-\cos 2x}{2}\,dx$ [using the identity cos 2x = 1 – 2sin²x] $=2\pi\left[x-\frac{\sin 2x}{2}\right]_0^{\pi}$ [Integrating] $=2\pi^2$. Thus each method of computation will yield a completely accurate result.	1 mark for $\int_0^{\pi}(2\sin x)^2\,dx$ If any errors are made in the above, award the last mark if the comment is sensible and consistent **Total = 8**	1 2 2 2 1

2ai	$\sqrt{(x-a)^2+y^2} = d_1$		1	[1]
2aii	$d_2 = \lvert x-b \rvert$	The absolute value is necessary	1	[1]
2aiii	$\left(\sqrt{(0-a)^2+0^2}\right)^2 = (\lvert 0-b \rvert)^2$ [As $d_1 = d_2$ when $x = 0$ and $y = 0$] hence $a^2 = b^2$ and as $a \neq b$ we must have $a = -b$		1 1 1	[3]
2aiv	$\left(\sqrt{(x-a)^2+y^2}\right)^2 = (\lvert x-b \rvert)^2$ hence $(x-a)^2+y^2 = (x-b)^2$ hence $x^2 - 2ax + a^2 + y^2 = x^2 - 2bx + b^2$ hence $-2ax + y^2 = -2bx$ [As $a^2 = b^2$] hence $y^2 = 2ax - 2bx = 4ax$ [As $-b = a$]	Nb: b could be replaced anywhere by -a $(x-a)^2+y^2 = (x-b)^2$	1 2	[3]
2av	The equation of the line is $x = a$ thus $y^2 = 4aa$ [By substitution] $= 4a^2$ hence $y = \pm 2a$ hence the points of intersection are $(a,2a)$ and $(a,-2a)$.	$y^2 = 4a^2$ **Total = 10**	1 1	[2]
2bi		1 for each graph	2	[2]
2bii	Implicit differentiation is applied to the first parabola: $\frac{dy}{dx} = \frac{2g}{y_1}$ and to the second parabola: $\frac{dy}{dx} = \frac{-2h}{y_1}$ Each $\frac{dy}{dx}$ gives the gradient of the tangent at the point (x_1,y_1). Perpendicular tangents $\Rightarrow$ the product of their gradients is -1 so $\frac{2g}{y_1} \times \frac{-2h}{y_1} = -1$ $\frac{-4gh}{{y_1}^2} = -1$ hence ${y_1}^2 = 4gh$ and so $4gx_1 = 4gh$ hence $x_1 = h$ and also $y_1 = \pm\sqrt{4gh}$		2 1 2 1	[6]
2biii	From the second parabola equation we have $y^2 = -4h(x-c)$ with $(b, \sqrt{4gh})$ lying on it thus $4gh = -4h(h-c)$ thus $g = -h + c$ [Dividing by 4h and expanding] thus $c = g + h$	**Total = 10**	1 1	[2]

3a	The area A of the triangle is $(x + a)y$ where (x,y) is a point lying on the ellipse in the first quadrant. Thus $A^2 = (x + a)^2y^2$ [Squaring]		
	Now $y^2 = b^2(1 - \frac{x^2}{a^2})$ [From the ellipse equation]		1
	Hence substituting yields $A^2 = b^2(x + a)^2(1 - \frac{x^2}{a^2})$	Note: Candidates may use	1
	Differentiating: $(A^2)' = b^2(2(x + a)(1 - \frac{x^2}{a^2}) - \frac{2x}{a^2}(x + a)^2)$	$A = b(a+x)\sqrt{1-\frac{x^2}{a^2}}$ to get	
	On simplification and factorisation we get $(A^2)' = 2b^2(x + a)(1 - \frac{x^2}{a^2} - \frac{x}{a^2}(x + a))$		
	Which changes to $(A^2)' = 2b^2(x + a)(1 - \frac{x}{a} - \frac{2x^2}{a^2}) = 2b^2(x + a)\frac{(a-2x)(a+x)}{a^2}$	$\frac{dA}{dx} = b\frac{a^2 - ax - 2x^2}{a(\sqrt{a^2 - x^2}}$	4
	For maximum $(A^2)' = 0$ so $(a-2x)(a+x) = 0$ thus $x = 0.5a$ hence $\frac{(0.5a)^2}{a^2} + \frac{y^2}{b} = 1$ [Substituting in the ellipse equation]		
	Thus $y^2 = 0.75b^2$		3
	Substituting for x and y in the area formula gives $A = \frac{3\sqrt{3}}{4}ab$ which is greater than the area of ½ ab obtained for the area of the triangle in the middle thus Hemi is right and Marsha wrong.	Or candidates can point out that max at $x = \frac{1}{2}a, a \neq 0$ so not where Q is on Y axis	2 **Total = 11**
3b	Using the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{b^2} = 1$ Sione should realise that $y^2 + z^2 = r^2$ where r is the radius of his cone.		1
	From the ellipsoid formula we have by substitution $\frac{x^2}{a^2} + \frac{r^2}{b^2} = 1$ thus $r^2 = b^2(1 - \frac{x^2}{a^2})$		1
	The volume of the cone is thus $V = \frac{1}{3}\pi(x + a)b^2(1 - \frac{x^2}{a^2})$ [Using the formula for volume of a cone $\frac{1}{3}\pi r^2 h$]		2
	Now $V' = \frac{1}{3}\pi b^2((1 - \frac{x^2}{a^2}) + (x+a)(-\frac{2x}{a^2}))$ Hence $V' = \frac{1}{3}\pi b^2\frac{(a-3x)(a+x)}{a^2}$		2
	Hence there will be maximum volume when $x = \frac{a}{3}$		1
	Which leads via substitution to $\frac{1}{9} + \frac{r^2}{b^2} = 1$ thus $r^2 = \frac{8b^2}{9}$		1
	Hence the maximum volume is $\frac{1}{3}\pi(\frac{a}{3} + a)\frac{8b^2}{9} = \frac{32\pi ab^2}{81}$		1 **Total = 9**

4a	$(cis\varphi)^3 = cis3\varphi + i\sin 3\varphi$ by De Moivre's theorem also $(cis\varphi)^3 = (\cos\varphi + i\sin\varphi)^3$ $= \cos^3\varphi + 3\cos^2\varphi\, i\sin\varphi + 3\cos\varphi(i\sin\varphi)^2 + (i\sin\varphi)^3$ $= \cos^3\varphi + 3i\cos^2\varphi\sin\varphi - 3\cos\varphi\sin^2\varphi - i\sin^3\varphi$ Equating imaginary parts: $\sin 3\varphi = 3\cos^2\varphi\sin\varphi - \sin^3\varphi$ $= 3(1-\sin^2\varphi)\sin\varphi - \sin^3\varphi$ using $\sin^2\varphi + \cos^2\varphi = 1$ $= 3\sin\varphi - 4\sin^3\varphi$		1 1 1 1 1	[5]
4bi	$z^n = 1 \Rightarrow z^n = cis(2k\pi)$ $\Rightarrow z = cis\left(\frac{2k\pi}{n}\right), \quad k = 0,\ldots\ldots,n-1$		1 1	[2]
4bii	$(1-z)^n = z^n \Rightarrow \left(\frac{1-z}{z}\right)^n = 1$ $\Rightarrow \frac{1-z}{z} = cis\left(\frac{2k\pi}{n}\right), \quad k = 0,\ldots\ldots,n-1$ Setting $\alpha_k = \frac{2k\pi}{n}$ $(1-z)^n = z^n \Rightarrow \frac{1-z}{z} = cis(\alpha_k), \quad k = 0,\ldots\ldots,n-1$ Making z the subject gives $z = \frac{1}{1+\cos\alpha_k + i\sin\alpha_k}$ $= \frac{1}{2\cos^2\frac{1}{2}\alpha_k + 2i\sin\frac{1}{2}\alpha_k\cos\frac{1}{2}\alpha_k}$ $= \frac{1}{2\cos\frac{1}{2}\alpha_k\left(\cos\frac{1}{2}\alpha_k + i\sin\frac{1}{2}\alpha_k\right)}$ $= \frac{1}{2\cos\frac{1}{2}\alpha_k}\left(cis\frac{1}{2}\alpha_k\right)^{-1}$ $= \frac{cis\frac{-1}{2}\alpha_k}{2\cos\frac{1}{2}\alpha_k}$ $= \frac{\cos\frac{1}{2}\alpha_k - i\sin\frac{1}{2}\alpha_k}{2\cos\frac{1}{2}\alpha_k}$ $= \frac{1}{2} - \frac{i}{2}\tan\frac{k\pi}{n}$	Note: It is not necessary to set $\alpha_k = \frac{2k\pi}{n}$, candidates may just carry the $\frac{2k\pi}{n}$ through **Total = 12**	2 3	[5]

4ci	Since $k>1$, $n+k>n+1$ Hence $\dfrac{1}{n+k}<\dfrac{1}{n+1}$	Accept any reasonable explanation	1 [1]
4cii	$\dfrac{1}{(n+1)}+\dfrac{1}{(n+1)(n+2)}+\dfrac{1}{(n+1)(n+2)(n+3)}+\ldots\ldots\ldots\ldots$ $<\dfrac{1}{n+1}+\dfrac{1}{(n+1)^2}+\dfrac{1}{(n+1)^3}+\ldots\ldots\ldots$ $=\dfrac{\frac{1}{n+1}}{1-\frac{1}{n+1}}$ $=\dfrac{1}{n+1-1}$ $=\dfrac{1}{n}$		1 1 1 [3]
4ciii	Let $\dfrac{m}{n}=1+\dfrac{1}{1!}+\dfrac{1}{2!}+\ldots\ldots\ldots+\dfrac{1}{n!}+\dfrac{1}{(n+1)!}+\dfrac{1}{(n+2)!}+\ldots\ldots\ldots$ So $\dfrac{m\,n!}{n}=n!+\dfrac{n!}{1!}+\dfrac{n!}{2!}+\ldots\ldots\ldots+\dfrac{n!}{n!}+\dfrac{n!}{(n+1)!}+\dfrac{n!}{(n+2)!}+\ldots\ldots\ldots$ Now the LHS and each of the terms of $n!+\dfrac{n!}{1!}+\dfrac{n!}{2!}+\ldots\ldots\ldots+\dfrac{n!}{n!}$ are whole numbers But $\dfrac{n!}{(n+1)!}+\dfrac{n!}{(n+2)!}+\ldots\ldots\ldots=\dfrac{1}{(n+1)}+\dfrac{1}{(n+1)(n+2)}+\ldots\ldots\ldots<\dfrac{1}{n}$ Thus we have: whole number = whole number + fraction !!!!!	**Total = 8**	1 1 1 1 [4]

5ai	$\text{RHS} = \dfrac{2\tan\frac{\varphi}{2}}{\sec^2\frac{\varphi}{2}} = \dfrac{2\sin\frac{\varphi}{2}}{\cos\frac{\varphi}{2}}\,\dfrac{\cos^2\frac{\varphi}{2}}{1}$ $= 2\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}$ $= \sin\varphi = \text{LHS}$		2 [2]
5aii	$I = \int \operatorname{cosec}\varphi\, d\varphi = \frac{1}{2}\int \dfrac{\sec^2\frac{\varphi}{2}}{\tan\frac{\varphi}{2}}\, d\varphi$ Set $u = \tan\frac{\varphi}{2}$ Then $du = \frac{1}{2}\sec^2\frac{\varphi}{2}\, d\varphi$ $I = \int \frac{du}{u}$ $= \ln\lvert u\rvert + c$ $= \ln\left\lvert\tan\frac{\varphi}{2}\right\rvert + c$	Accept also differentiation of RHS	3 [3]
5aiii	$I = \int \dfrac{\sqrt{1-y^2}}{y}\, dy$ Set $y = \sin\varphi$ $dy = \cos\varphi\, d\varphi$ $\sqrt{1-y^2} = \sqrt{1-\sin^2\varphi} = \cos\varphi$ $I = \int \dfrac{\cos^2\varphi}{\sin\varphi}\, d\varphi$ $= \int \dfrac{1-\sin^2\varphi}{\sin\varphi}\, d\varphi$ $= \int (\operatorname{cosec}\varphi - \sin\varphi)\, d\varphi$ $I = \ln\left\lvert\tan\frac{\varphi}{2}\right\rvert + \cos\varphi + c$ $\cos\varphi = \sqrt{1-\sin^2\varphi} = \sqrt{1-y^2}$ $\sin\varphi = \dfrac{2\tan\frac{\varphi}{2}}{\sec^2\frac{\varphi}{2}}$ so $\tan\frac{\varphi}{2} = \dfrac{\sin\varphi}{2\cos^2\frac{\varphi}{2}} = \dfrac{\sin\varphi}{1+\cos\varphi} = \dfrac{y}{1+\sqrt{1-y^2}}$ So $I = \sqrt{1-y^2} + \ln\left\lvert\dfrac{y}{1+\sqrt{1-y^2}}\right\rvert + c$	Do not insist on absolute value **Total = 11**	2 1 1 1 1 [6]

5b	Line is $\frac{y-b}{x-a}=m$ Cuts Ox when $y=0$ $\frac{-b}{x-a}=m \Rightarrow x=-\frac{b}{m}+a$		1 [1]
5ci	By 5b, the coordinates of D are $\left(\frac{-y}{y'}+x,0\right)$		1 [1]
5cii	$JN^2+ND^2=JD^2$ $y^2+\left(\frac{y}{y'}\right)^2=1$ Take –ve root as y is decreasing wrt x $\frac{y}{y'}=-\sqrt{1-y^2}$	Note, for the second mark the candidate must explain –ve sign	2 [2]
5ciii	$\frac{dy}{dx}=\frac{-y}{\sqrt{1-y^2}}$ $\int\frac{\sqrt{1-y^2}}{y}dy=-\int dx$ $x=-\sqrt{1-y^2}-\ln\left\|\frac{y}{1+\sqrt{1-y^2}}\right\|+c$ When $x=0$ $y=1$ $0=0-\ln 1+c \Rightarrow c=0$ $x=-\sqrt{1-y^2}-\ln\left(\frac{y}{1+\sqrt{1-y^2}}\right)$ as $y>0$	This step must be performed ie evaluating c Don't penalise if absolute value left .	1 1 1 1 [4]
5ciii	Approximately 6.6m	**Total = 9**	1 [1]

6ai	$\cos\varphi = \cos\left(\frac{\varphi}{2}+\frac{\varphi}{2}\right)$ $= \cos\frac{\varphi}{2}\cos\frac{\varphi}{2} - \sin\frac{\varphi}{2}\sin\frac{\varphi}{2}$ $= \cos^2\frac{\varphi}{2} - \sin^2\frac{\varphi}{2}$ $= 1 - \sin^2\frac{\varphi}{2} - \sin^2\frac{\varphi}{2}$ $= 1 - 2\sin^2\frac{\varphi}{2}$		2 [2]
6aii	When φ is small, $\sin\frac{\varphi}{2} \approx \frac{\varphi}{2}$ So $\cos\varphi \approx 1 - 2\left(\frac{\varphi}{2}\right)^2 = 1 - \frac{\varphi^2}{2}$	**Total = 4**	2 [2]
6b	$x = r\cos\varphi$ $y = r\sin\varphi$	**Total = 1**	[1]
6ci	By the cosine rule		1 [1]
6cii	$\delta s^2 = (r+\delta r)^2 + r^2 - 2r(r+\delta r)\cos(\delta\varphi)$ $= (r+\delta r)^2 + r^2 - 2r(r+\delta r)\left(1 - \frac{\delta\varphi^2}{2}\right)$ $= r^2 + 2r\delta r + \delta r^2 + r^2 - 2r^2 - 2r\delta r + 2r^2\frac{\delta\varphi^2}{2} - 2r\delta r\frac{\delta\varphi^2}{2}$ $\approx \delta r^2 + r^2\delta\varphi^2$ Ignore the $-2r\delta r\frac{\delta\varphi^2}{2}$ term as it involves a product of 3 small quantities, unlike the other terms which involve only 2.	Any reasonable explanation **Total = 4**	2 1 [3]
6di	This way the police boat will always be at the same distance from the origin as the other boat.		1 [1]
6dii	$v^2 = \left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\varphi}{dt}\right)^2$ $= \left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{dr}{dt}\frac{d\varphi}{dr}\right)^2$ $= \left(\frac{dr}{dt}\right)^2\left(1 + r^2\left(\frac{d\varphi}{dr}\right)^2\right)$ $2500 = 900\left(1 + r^2\left(\frac{d\varphi}{dr}\right)^2\right)$ as $\frac{dr}{dt} = 30$ by 6di $\frac{25}{9} = 1 + r^2\left(\frac{d\varphi}{dr}\right)^2$ $\frac{16}{9} = r^2\left(\frac{d\varphi}{dr}\right)^2$	1 mark for using $\frac{dr}{dt} = 30$	1 2 1 [4]

6dii	$r\dfrac{d\varphi}{dr}=\dfrac{4}{3}$ Take + root as $\dfrac{d\varphi}{dr}$ is positive (φ increases with r) $\int\dfrac{dr}{r}=\dfrac{3}{4}\int d\varphi$ $\ln\|r\|=\dfrac{3}{4}\varphi+c$ $r=Ae^{\frac{3}{4}\varphi}$ When $r=1$ take $\varphi=0$ so that $A=1$ $r=e^{\frac{3}{4}\varphi}$	Don't penalise if reason not given	1 1 1 [3]
6diii	A spiral.		1 [1]
6div	Head back towards the origin or stop or any sensible strategy backed by a reason.	1 for strategy, 1 for reason **Total = 11**	2 [2]