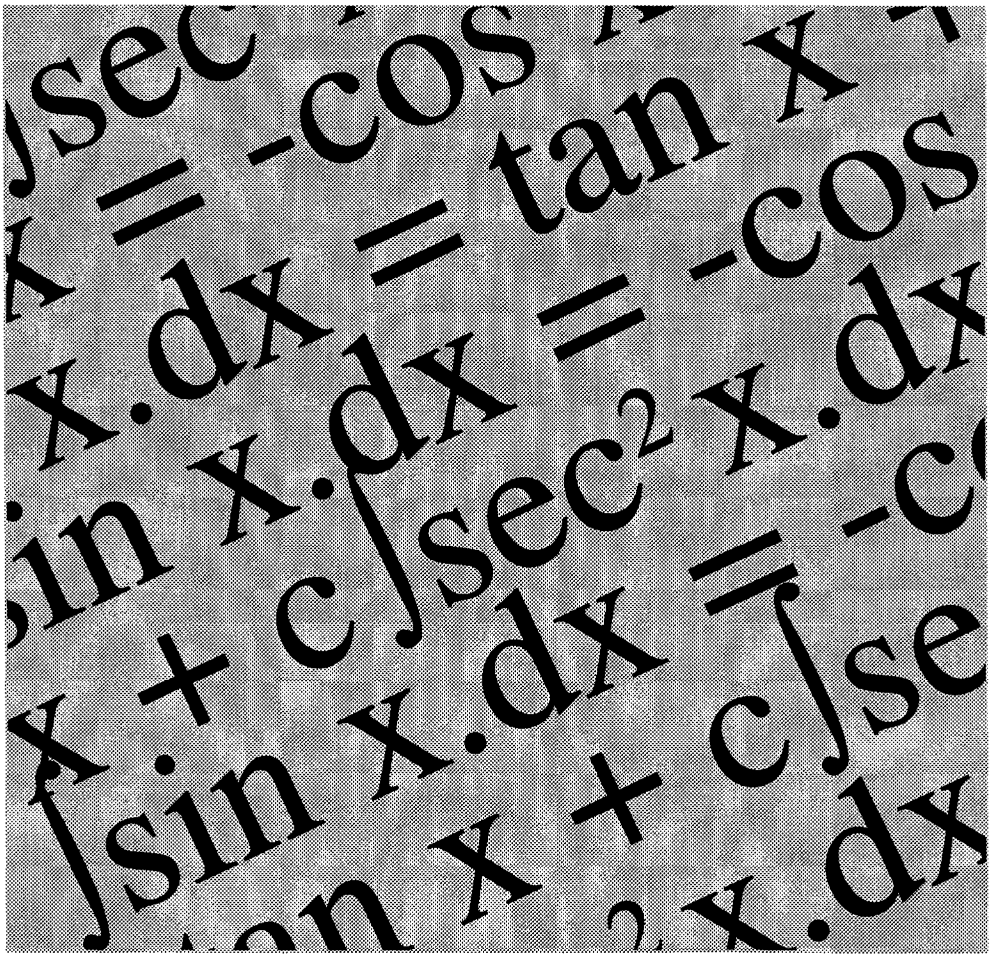
NZEST SCHOLARSHIP EXAMINATION

2002 EXAMINER'S REPORT
AND MARKING GUIDELINES

MATHEMATICS WITH CALCULUS



NZEST

NZEST EXAMINER'S REPORT

2002 NZEST Mathematics with Calculus

Mark Adjusting
Although NZEST does not practise scaling there is a necessity to adjust the calculus marks as there is a total of 120 marks available to the candidates and 5 scored more than 100 with one candidate getting a score of 117, a most remarkable achievement.

The marks were adjusted by using a piecewise function as follows:

If a candidate scored 90 or less then their mark was not adjusted.

If the candidate scored 91 or more then their new mark was the rounded value of $90+(x-90)\div 3$ where x was the mark they got out of 120.

General Comments
It was agreed by the markers that this was a challenging paper with enough in it for most candidates to attempt without any part being too easy. The paper challenged the most able students without any impossibly difficult questions. The raw statistics for the adjusted marks are given below for a large sample of students.

Statistic	*Value*
Minimum	1
Lower Quartile	30
Median	36
Mean	44
Upper Quartile	63
Maximum	100
Standard Deviation	22

Question 1

A good question to start the paper. The median obtained was 8.5 out of 20 with a lowest mark of 0 obtained by 38 of the 550 sampled who attempted this question. The highest mark was 20 which was obtained by 22 candidates. Full statistics of this sample are given below.

Statistic	*Value*
Minimum	0
Lower Quartile	4
Median	8.5
Upper Quartile	14
Maximum	20
Standard Deviation	6.13

1(a)(i)
A common error was $R^2 = h^2 + r^2$

Some students put the formula for V in terms of r and consequently made the differentiation more complicated.

Many students missed out on the second derivative test.

(ii) Some students failed to simplify the ratio.

(b)(i) Usually well done.
(ii) Many failed to use b(i) to find the result and tried to use some heavy algebra as a result.

It is clear that applications such as those in Q1 need more practice among the able students as a median of 9/20 is a bit low for what were two straightforward applications.

Question 2
Only six students of the many who attempted this question got full marks. 180 failed to get more than 0.

Many managed a page of algebra only to arrive back at the original equation

It was disappointing for the markers to see able students unable to use the technique of separation of variables.

Other common errors were assuming the arbitrary constant was 0 or just ignoring it. Some named their constant K and confused it later with the parameter k in the question.

Many ignored the instructions in (a)(i) and solved the differential equation by other methods.

Question 3
The statistics for this question from a sample of over 500 are below:

Statistic	*Value*
Minimum	0
Lower Quartile	2
Median	7
Upper Quartile	11
Maximum	20
Standard Deviation	5.8

The markers of this question gave a good summary of the different parts of this question. Overall, students found the steps of the proof difficult but there were parts such as b(i), b(ii) and c(ii) which were answered well.

Question 4

This question was not well done with a median of 3.

(a)(i) The key to this question was the simplification $\frac{2\alpha}{2^n} = \frac{\alpha}{2^{n-1}}$.

(ii) Most candidates did not start from cos 2_ but from the square root expression on the left hand side.

(b) (i) It is interesting that half of the candidates who attempted this question started out by writing an inequality for lengths rather than areas as was asked in the question.

A minority of candidates were able to get to part (c) of this question. Provided instructions were followed a successful conclusion to this question usually ensued.

It is interesting that there were some innovative but correct approaches which were different to that suggested by the examiner.

Question 5

The median for this question was 5 and the mean mark was 5.5. The lower quartile was 2 and the upper quartile was 8. The mode was 5 and there was a full range of marks from 0 to 20. Sixteen students did not attempt this question.

Interpretation of what was being asked proved a major problem and indicates that practice in problems of this type needs to be done.

(a)This part of the question which involved making standard sketches of an ellipse and a hyperbola was done well.

(b)(i)Those who differentiated implicitly generally gained full marks. A very few candidates used parametric equations . Any evidence of $m_1m_2 = -1$ gained a mark.

(b)(ii)Interpretation of what this question meant seemed a major problem. Many of the successful candidates used an alternative strategy which proved an easy way of obtaining six marks.

(c) Many candidates started with an assumption that the curves met at right angles. If this assumption was made then a maximum of only one mark was obtainable.

(d)(i) Most attempting this reworded the question as their answer rather than relating it to the asymptotes of the hyperbola.

(d)(ii) and (iii) If candidates got this far they generally finished the question well.

Question 6

A sample of the papers of 350 candidates was taken and the following statistics were obtained.

Statistic	*Value*
Minimum	0
Lower Quartile	2
Median	6
Upper Quartile	10
Maximum	19

Only about fifteen students did not attempt this question.

Marker's comments on the individual portions of this question follow:

(a)(i) The expansion was quite well done but correctly applying it to obtain the correct expression proved difficult for many.

(a)(ii) This was very difficult for most although there were some very concise expressions from the more able candidates.

(b)(i) Generally well done but spoilt too often by basic errors.

(b)(ii) A general unfamiliarity with the attempting of a proof was the main problem here. Candidates need to ensure that proofs are complete and that necessary working is not omitted.

(b)(iii) There was a problem in the wording of this question which led many candidates and at least two markers to consider an alternative. Although this permitted solutions for most of this part of the question it made (b)(iii)(3) virtually impossible.

A majority of candidates attempted this part of question six and interpreted it the way the examiner had intended. There were some very good attempts from the better candidates.

Comments from schools:

Eleven schools with a number of candidates were asked to comment about the paper. The table below summarises these comments.

Was the paper sufficiently challenging ?	Six schools thought so. At least two thought there was not enough material for the mid level and one school was quite scathing.
Were there any inappropriate questions ?	The only questions named by any school were Q3 and Q4. Most schools did not see any question as inappropriate.
Was the coverage of the syllabus satisfactory ?	Nine thought so but two schools did not.
Was the paper of the right length ?	Only three schools thought the paper was of the right length with most feeling that the paper was too long.
Possible improvements ?	More material for mid level students.
	More effort made to raise the average of this exam.
	Shorten slightly.
	Ensure students can get started.

The responses from schools are very interesting and will provide food for thought for this coming year's examination setters.

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NZEST ANSWERS

MATHEMATICS WITH CALCULUS

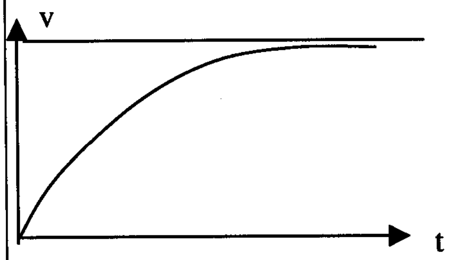
2002

Question 1

a.(i)	$V = \pi r^2 h$ $R^2 = r^2 + \frac{h^2}{4}$ $V = \pi(R^2 - \frac{h^2}{4})h$ $= \pi R^2 h - \pi \frac{h^3}{4}$	1	
	$V' = \pi R^2 - \frac{3\pi h^2}{4}$ $= 0$	1	
	$h^2 = \frac{4R^2}{3} \qquad h = \frac{2R}{\sqrt{3}}$	1	
	$R^2 = r^2 + \frac{R^2}{3} \Rightarrow r^2 = \frac{2R^2}{3} \qquad r = \sqrt{\frac{2}{3}}\,R$	1	
	$V'' = -\frac{6\pi h}{4} < 0$ ie max $\boxed{5}$	1	
a.(ii)	$\frac{V_{cyl}}{V_{sphere}} = \frac{\pi r^2 h}{4/3\pi R^3} = \frac{3r^2 h}{4R^3}$	1	
	$= \frac{3}{4R^3} \times \frac{2R^2}{3} \times \frac{R}{\sqrt{3}}$ ie $1 : \sqrt{3}$ or $\sqrt{3} : 3$ $\boxed{2}$	1	

b(i)	$\cos\alpha = \dfrac{x}{\sqrt{400+x^2}}$ $\quad\quad \cos\beta = \dfrac{20-x}{\sqrt{400+(20-}$ **2**	2	
b.(ii)	$AP = \sqrt{20^2 + x^2}$ $PB = \sqrt{20^2 + (20-x)^2}$ $t_{AP} = \dfrac{\sqrt{400+x^2}}{100}$ $t_{PB} = \dfrac{\sqrt{400+(20-x)^2}}{50}$ $t_{AB} = \dfrac{\sqrt{400+x^2}}{100} + \dfrac{\sqrt{400+(20-x)^2}}{50}$ $t_{AB}' = \dfrac{1}{100}\times\dfrac{2x}{2\sqrt{400+x^2}} + \dfrac{1}{50}\times\dfrac{2(20-x).-1}{2\sqrt{400+(20-x)^2}}$ $t_{AB}' = \dfrac{x}{100\sqrt{400+x^2}} - \dfrac{(20-x)}{50\sqrt{400+(20-x)^2}}$ $= \dfrac{1}{100}\cos\alpha - \dfrac{1}{50}\cos\beta$ $= 0$ $\cos\alpha = 2\cos\beta$ **11**	1 1 1 1 1 3 1 1 1	

Question 2

a(i)	$\dfrac{mdv}{10m-kv}=dt$ $m\displaystyle\int\frac{dv}{10m-kv}=\int dt$ $\dfrac{-m}{k}\ln\lvert 10m-kv\rvert=t+c$ $\ln\lvert 10m-kv\rvert=\dfrac{-k}{m}t+C$ $\lvert 10m-kv\rvert=e^{\frac{-k}{m}t+C}$ $=e^{\frac{-k}{m}t}e^{C}$ $10m-kv=Ae^{\frac{-k}{m}t}$ $v(0)=0$ $10m=A$ $v=\dfrac{10m}{k}\left(1-e^{\frac{-k}{m}t}\right)$ $A=\dfrac{10m}{k}\quad B=\dfrac{k}{m}$ $\boxed{9}$	1 3 3 1 1	 1 LHS 1 Abs value 1 RHS Use of log constant OK
a(ii) $\dfrac{10m}{k}$	 Mary's speed approaches a limiting value of $\dfrac{10m}{k}$. It does not increase indefinitely. $\boxed{3}$	3	1 Rough shape 1 asymptote & value 1 conclusion

b(i)	$\frac{dx}{dt}=\frac{10m}{k}\left(1-e^{\frac{-k}{m}t}\right)$	1	
	$x=\frac{10m}{k}\left(t+\frac{m}{k}e^{\frac{-k}{m}t}\right)+c$	2	Don't penalise +c missing
	$x(0)=0$ $0=\frac{10m^2}{k^2}+c$		
	$c=\frac{-10m^2}{k^2}$	1	
	$x=\frac{10m}{k}\left(t+\frac{m}{k}e^{\frac{-k}{m}t}-\frac{m}{k}\right)$ **5**	1	
b(ii)	As t increases $x\rightarrow\frac{10m}{k}\left(t-\frac{m}{k}\right)$ which is a	1	1 e terms→0 (or implied)
	straight line of gradient $\frac{10m}{k}$.	1	1 straight line & gradient
	This means the distance is increasing at a constant rate. ie the velocity is constant **3**	1	1 conclusion

Question 3

a.(i)	$(x\log_e x - x)' = x\frac{1}{x} + \log_e x - 1$ $= \log_e x$ $\int_1^n \log_e x = [x\log_e x - x]_1^n$ $= (n\log_e n - n) - (0-1)$ $= n\log_e n + 1 - n$ $\boxed{2}$	1 1	
a(ii)	$\int_1^n \log_e x = \frac{1}{2}(\log_e 1 + 2(\log_e 2 + ... + \log_e(n-1)) + \log_e n)$ $= \log_e 2.......(n-1) + \frac{1}{2}\log_e n$ $= \log_e 2.......n - \frac{1}{2}\log_e n$ $= \log_e n! - \frac{1}{2}\log_e n$ $\boxed{3}$	1 1 1	 Sum to product factorial
b(i)	$\int_1^n \log_e x == n\log_e n + 1 - n$ $= \log_e n^n + \log_e(e^{1-n})$ $= \log_e(n^n e^{1-n})$ $\boxed{2}$	 1 1	 for changing to $\log_e$
b(ii)	Trapezium rule gives $\log_e n! - \log_e n^{1/2} = \log_e(n!n^{-1/2})$ So $n!n^{-1/2} \approx n^n e^{1-n}$ $n! \approx n^{n+1/2}e^{1-n}$ $\boxed{3}$	1 1 1	
c.(i)	Area $= \frac{1}{2}\log_e\frac{5}{4} + \log_e 2 + \log_e 3 + ... + \log_e(n-1) + \frac{1}{2}\log_e n$ $= \frac{1}{2}\log_e\frac{5}{4} + \log_e(n-1)! + \frac{1}{2}\log_e n$ Diff $= \frac{1}{2}\log_e\frac{5}{4} + \log_e(n-1)! + \frac{1}{2}\log_e n - (\log_e(n-1)! + \frac{1}{2}\log_e n)$ $= \frac{1}{2}\log_e\frac{5}{4}$ $\boxed{4}$	1 1 1 1	
c.(ii)	The first area is always less than the area under the curve and the second area is always greater. $\boxed{1}$	1	

c.(iii)	$0<\int_1^n \log_e x\,dx-(\log_e n!-\frac{1}{2}\log_e n)<\frac{1}{2}\log_e\frac{5}{4}$ $0<\log_e(n^n e^{1-n})-\log_e(n!n^{-1/2})<\frac{1}{2}\log_e\frac{5}{4}$ $-\log_e(n^n e^{1-n})<-\log_e(n!n^{-1/2})<\frac{1}{2}\log_e\frac{5}{4}-\log_e(n^n e^{1-n})$ $-\frac{1}{2}\log_e\frac{5}{4}+\log_e(n^n e^{1-n})<\log_e(n!n^{-1/2})<\log_e(n^n e^{1-n})$ $\log_e\left(\sqrt{\frac{4}{5}}n^n e^{1-n}\right)<\log_e(n!n^{-1/2})<\log_e(n^n e^{1-n})$ $\sqrt{\frac{4}{5}}n^n e^{1-n}<n!n^{-1/2}<n^n e^{1-n}$ $\sqrt{\frac{4}{5}}n^{n+1/2}e^{1-n}<n!<n^{n+1/2}e^{1-n}$ $\sqrt{\frac{4}{5}}e^{1-n}n^{n+1/2}<n!<e^{1-n}n^{n+1/2}$ $\boxed{5}$	1 1 1 1 1	inequality Log monotonically increasing means log can be removed.*

* Give bonus mark here if reason is mentioned (ie monotonic increase) unless total is 20.

Question 4

a.(i)	$\sin\frac{\alpha}{2^n}\cos\frac{\alpha}{2^n}=\frac{1}{2}\sin\frac{2\alpha}{2^n}$ $=\frac{1}{2}\sin\frac{\alpha}{2^{n-1}}$ $\boxed{1}$	1	
a.(ii)	$\cos\alpha=2\cos^2\frac{\alpha}{2}-1$ $1+\cos\alpha=2\cos^2\frac{\alpha}{2}$ $\sqrt{\frac{1}{2}+\frac{1}{2}\cos\alpha}=\cos\frac{\alpha}{2}$ $\boxed{1}$	1	
b.(i)	Area ΔAPB $\leq$ Area sector APB $\leq$ AreaΔAPC $\frac{1}{2}\times1^2\times\sin x\leq\frac{1}{2}\times1^2\times x\leq\frac{1}{2}\times1\times\tan x$ $\sin x\leq x\leq\tan x$ $1\leq\frac{x}{\sin x}\leq\frac{1}{\cos x}$ $\boxed{2}$	1 1	
b.(ii)	$1\leq\frac{\alpha/2^n}{\sin(\alpha/2^n)}\leq\frac{1}{\cos(\alpha/2^n)}$ $\boxed{3}$ $\frac{1}{\alpha}\leq\frac{1/2^n}{\sin(\alpha/2^n)}\leq\frac{1}{\alpha}\times\frac{1}{\cos(\alpha/2^n)}$ $\lim\limits_{n\to\infty}\frac{1/2^n}{\sin(\alpha/2^n)}=\frac{1}{\alpha}$ since $\lim\limits_{n\to\infty}\cos\left(\frac{\alpha}{2^n}\right)=\cos 0=1$	1 1 1	
c.(i)	$\cos\frac{\alpha}{2}\cos\frac{\alpha}{4}\cos\frac{\alpha}{8}\ldots\ldots\ldots\ldots\cos\frac{\alpha}{2^{n-1}}\cos\frac{\alpha}{2^n}\sin\frac{\alpha}{2^n}$ $=\cos\frac{\alpha}{2}\cos\frac{\alpha}{4}\cos\frac{\alpha}{8}\ldots\ldots\ldots\ldots\cos\frac{\alpha}{2^{n-1}}\left(\cos\frac{\alpha}{2^n}\sin\frac{\alpha}{2^n}\right)$ $=\frac{1}{2}\cos\frac{\alpha}{2}\cos\frac{\alpha}{4}\cos\frac{\alpha}{8}\ldots\ldots\ldots\ldots\cos\frac{\alpha}{2^{n-1}}\sin\frac{\alpha}{2^{n-1}}$ $=\frac{1}{4}\cos\frac{\alpha}{2}\cos\frac{\alpha}{4}\cos\frac{\alpha}{8}\ldots\ldots\ldots\ldots\sin\frac{\alpha}{2^{n-2}}$ $=\ldots\ldots$ $=\frac{1}{2^{n-2}}\cos\frac{\alpha}{2}\cos\frac{\alpha}{4}\sin\frac{\alpha}{4}$ $\boxed{5}$ $=\frac{1}{2^{n-1}}\cos\frac{\alpha}{2}\sin\frac{\alpha}{2}$ $=\frac{1}{2^n}\sin\alpha$	1 1 1 1 1	for these 2 steps pattern for same 2 steps at end

c.(ii)	$\cos\frac{\alpha}{2}\cos\frac{\alpha}{4}\cos\frac{\alpha}{8}\ldots\ldots = \lim_{n\to\infty}\frac{1}{2^n}\frac{\sin\alpha}{\sin\frac{\alpha}{2^n}}$	1	
	$= \sin\alpha\lim_{n\to\infty}\frac{1/2^n}{\sin\frac{\alpha}{2^n}}$	1	
	$= \frac{\sin\alpha}{\alpha}$	1	
	$\alpha = \frac{\sin\alpha}{\cos\frac{\alpha}{2}\cos\frac{\alpha}{4}\cos\frac{\alpha}{8}\ldots\ldots}$ $\boxed{4}$	1	
c.(iii)	$\alpha = \frac{1}{\cos\frac{\alpha}{2}\sqrt{\frac{1}{2}+\frac{1}{2}\cos\frac{\alpha}{2}}\sqrt{\frac{1}{2}+\frac{1}{2}\cos\frac{\alpha}{4}}\ldots\ldots}$	1	OK to put $\pi/2$ in here
	$= \frac{1}{\cos\frac{\alpha}{2}\sqrt{\frac{1}{2}+\frac{1}{2}\cos\frac{\alpha}{2}}\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}+\frac{1}{2}\cos\frac{\alpha}{2}}}\ldots\ldots}$	1	
	$\frac{\pi}{2} = \frac{1}{\cos\frac{\pi}{4}\sqrt{\frac{1}{2}+\frac{1}{2}\cos\frac{\pi}{4}}\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}+\frac{1}{2}\cos\frac{\pi}{4}}}\ldots\ldots}$	1	Substitute
	$= \frac{1}{\sqrt{\frac{1}{2}}\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}}}\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}}}}\ldots\ldots}$ $\boxed{4}$	1	answer

Note: Some method starting like the following is also acceptable for c(iii)

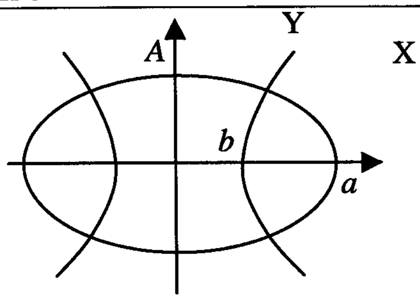
$\alpha = \frac{\pi}{2}$

$\cos\frac{\alpha}{2} = \frac{1}{\sqrt{2}}$

$\cos\frac{\alpha}{4} = \sqrt{\frac{1}{2}+\frac{1}{2\sqrt{2}}}$

etc

Question 5

a	 $\boxed{1}$	1	
b(i)	Ellipse grad : $\dfrac{-A^2x_0}{a^2y_0}$ Hyperbola grad : $\dfrac{B^2x_0}{b^2y_0}$ The curves intersect at right angles provided the product of gradients =-1 $\dfrac{-A^2x_0}{a^2y_0}\dfrac{B^2x_0}{b^2y_0}=-1$ $\dfrac{A^2B^2{x_0}^2}{a^2b^2{y_0}^2}=1$ $\boxed{4}$	2 1 1	
b(ii)	Since (x_0, y_0) lies on the ellipse: $\dfrac{{x_0}^2}{a^2}+\dfrac{{y_0}^2}{A^2}=1$ It also lies on the hyperbola so $\dfrac{{x_0}^2}{b^2}-\dfrac{{y_0}^2}{B^2}=1$ Solving for x_0, y_0 gives ${x_0}^2=\dfrac{a^2b^2(A^2+B^2)}{b^2A^2+a^2B^2}$, ${y_0}^2=\dfrac{A^2B^2(a^2-b^2)}{b^2A^2+a^2B^2}$ $\dfrac{{x_0}^2}{{y_0}^2}=\dfrac{a^2b^2(A^2+B^2)}{b^2A^2+a^2B^2}\times\dfrac{b^2A^2+a^2B^2}{A^2B^2(a^2-b^2)}$ $=\dfrac{a^2b^2(A^2+B^2)}{A^2B^2(a^2-b^2)}$ $=\dfrac{a^2b^2}{A^2B^2}$ by the assumption $\boxed{6}$ This leads to the condition in b(i)	3 2 1	multiplication and simplification **by assumption**
c	Since $A^2=a^2(1-e^2)=a^2-k^2$ and $B^2=b^2(f^2-1)=k^2-b^2$ $b^2+B^2=k^2$ $a^2-A^2=k^2$ $\boxed{3}$ Hence the curves meet at right angles as they satisfy the condition	1 1 1	

d(i)	The rocket will be travelling closely along one of asymptotes of the hyperbola $\boxed{1}$	1	
d(ii)	$y=\frac{B}{b}x$ $\frac{B^2}{b^2}=\frac{b^2(f^2-1)}{b^2}$ $=\frac{a^2e^2-b^2}{b^2}$ $=\frac{a^2e^2}{b^2}-1$ $y=\sqrt{\frac{a^2e^2}{b^2}-1}\ x$ $\boxed{3}$	1 1 1	
d(iii)	If $a=2b$ $y=\sqrt{4e^2-1}\ x$ $\frac{x^2}{a^2}+\frac{(4e^2-1)x^2}{a^2(1-e^2)}=1$ $x^2(1-e^2)+(4e^2-1)x^2=A^2$ $3e^2x^2=A^2$ $x=\frac{A}{\sqrt{3}e},\qquad y=\frac{A\sqrt{4e^2-1}}{\sqrt{3}e}$ $\boxed{2}$	1 1	

Question 6

a.(i)	$\left(x^m+\frac{1}{x^m}\right)\left(x^n+\frac{1}{x^n}\right)$ $=\left(x^{m+n}+\frac{1}{x^{m+n}}\right)+\left(x^{m-n}+\frac{1}{x^{m-n}}\right)$ So $x^{m+n}+\frac{1}{x^{m+n}}$ $=\left(x^m+\frac{1}{x^m}\right)\left(x^n+\frac{1}{x^n}\right)-\left(x^{m-n}+\frac{1}{x^{m-n}}\right)$ Thus $x^{m+1}+\frac{1}{x^{m+1}}$ $=\left(x^m+\frac{1}{x^m}\right)\left(x+\frac{1}{x}\right)-\left(x^{m-1}+\frac{1}{x^{m-1}}\right)$ $x^{2m}+\frac{1}{x^{2m}}=\left(x^m+\frac{1}{x^m}\right)^2-2$ $\boxed{3}$	1 1 1	
a.(ii)	$\left(x^3+\frac{1}{x^3}\right)$ $=\left(x^2+\frac{1}{x^2}\right)\left(x+\frac{1}{x}\right)-\left(x+\frac{1}{x}\right)$ $=a^3-3a$ $\left(x^6+\frac{1}{x^6}\right)$ $=\left(x^3+\frac{1}{x^3}\right)^2-2$ $=(a^3-3a)^2-2$ $\left(x^{12}+\frac{1}{x^{12}}\right)$ $=\left(x^6+\frac{1}{x^6}\right)^2-2$ $=\left((a^3-3a)^2-2\right)^2-2$	1 1	Note: This can also be done by working backwards 1st grouping next 2 groupings

	$\left(x^{24}+\dfrac{1}{x^{24}}\right)$ $=\left(x^{12}+\dfrac{1}{x^{12}}\right)^2-2$ $=\left(\left((a^3-3a)^2-2\right)^2-2\right)^2-2$ $\boxed{3}$	1	Answer
b.(i)(1)	$(a+b)^2(a-b)^3$ $=(a^2-b^2)^2(a-b)$ $=(a-b)(a^4-2a^2b^2+b^4)$ $=a^5-2a^3b^2+ab^4-a^4b+2a^2b^3-b^5$ $=a^5-a^4b-2a^3b^2+2a^2b^3+ab^4-b^5$ $\boxed{2}$	1 1	Must have this grouping
b.(ii)(1)	$\dfrac{1}{z}=z^{-1}$ So $\dfrac{1}{z}=(\cos\theta+i\sin\theta)^{-1}$ $=\cos-\theta+i\sin-\theta$ $=\cos\theta-i\sin\theta$ $\boxed{1}$	1	
b.(ii)(2)	$z^n=cisn\theta$ $=\cos n\theta+i\sin n\theta$ $\dfrac{1}{z^n}=(cis\theta)^{-n}$ $=cis(-n\theta)$ $=\cos-n\theta+i\sin-n\theta$ $=\cos n\theta-i\sin n\theta$ $z^n+\dfrac{1}{z^n}=2\cos n\theta$ $\boxed{2}$	1 1	
b.(ii)(3)	$z^n-\dfrac{1}{z^n}=2i\sin n\theta$ $\boxed{1}$	1	

b.(iii)(1)	$\left(z+\frac{1}{z}\right)^2\left(z-\frac{1}{z}\right)^3$ $= z^5 - z^4\frac{1}{z} - 2z^3\frac{1}{z^2} + 2z^2\frac{1}{z^3} + z\frac{1}{z^4} - \frac{1}{z^5}$ $= z^5 - z^3 - 2z + \frac{2}{z} + \frac{1}{z^3} - \frac{1}{z^5}$ $= \left(z^5 - \frac{1}{z^5}\right) - \left(z^3 - \frac{1}{z^3}\right) - 2\left(z - \frac{1}{z}\right)$ $= 2i\sin 5\theta - 2i\sin 3\theta - 4i\sin\theta$ As $i^3 = -i$ $-4\cos^2\theta\, 8\sin^3\theta\, i = 2i\sin 5\theta - 2\, i\sin 3\theta - 4i\sin$ $\cos^2\theta\sin^3\theta = -\frac{1}{16}\sin 5\theta + \frac{1}{16}\sin 3\theta + \frac{1}{8}\sin\theta$ $\boxed{5}$	1 1 1 1 1	
b.(iii)(2)	As LHS is even and RHS would be odd. Also accept $\cos^2\theta\sin^0\theta = \cos^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$ $= \frac{1}{2}\cos 0\theta - \frac{1}{2}\cos 2\theta$ $\boxed{1}$	1	Accept valid alternative explanations
b.(iii)(3)	The two possibilities are expansion in cosines of multiples of the angle, or expansions in sines. The cos expansion holds if the LHS is even and the sin expansion if it is odd. This is because you can't have odd function = even function. $\boxed{2}$	1 1	