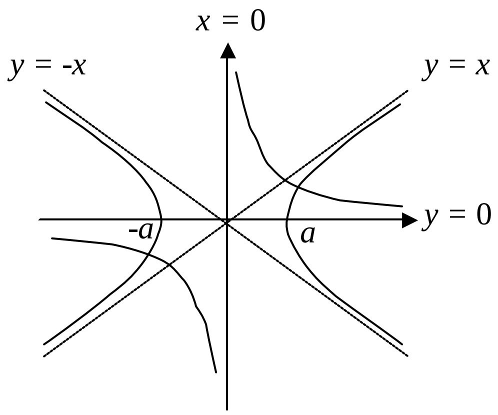
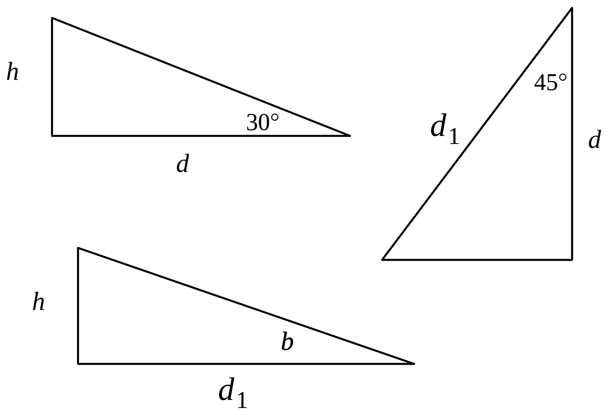
QUESTION 1			
1a (i)	$h = \dfrac{A - 2pr^2}{2pr}$		1
1a (ii)	$V = pr^2 h = pr^2 \dfrac{A-2pr^2}{2pr} = \dfrac{1}{2}Ar - pr^3$		1
1a (iii)	$\dfrac{dV}{dr} = \dfrac{1}{2}A - 3pr^2$ $\dfrac{dV}{dr} = 0 \Rightarrow r^2 = \dfrac{A}{6p}$ or $r = \sqrt{\dfrac{A}{6p}}$		3
1a (iv)	$\dfrac{d^2V}{dr^2} = -6pr < 0$ hence a maximum		2
1a (v)	$A = 2pr^2 + 2prh$ $r^2 = \dfrac{A}{6p} \Rightarrow \mathrm{A} = 6p\mathrm{r}^2$ $6pr^2 = 2pr^2 + 2prh$ $h = 2r$		3 **a =10**
1b (i)	$s + r = \dfrac{A}{pr} \qquad s - r = \dfrac{A}{pr} - 2r = \dfrac{A-2pr^2}{pr}$ $h^2 = s^2 - r^2 = (s-r)(s+r) = \dfrac{A}{pr} \times \dfrac{A-2pr^2}{pr}$ $= \dfrac{A(A-2pr^2)}{p^2r^2}$ $h = \dfrac{\sqrt{A(A-2pr^2)}}{pr}$		3
1b (ii)	$V = \dfrac{1}{3}pr^2 \dfrac{\sqrt{A(A-2pr^2)}}{pr} = \dfrac{\sqrt{A}}{3}\sqrt{Ar^2 - 2pr^4}$ $\dfrac{dV}{dr} = \dfrac{\sqrt{A}}{3} \dfrac{2Ar - 8pr^3}{2\sqrt{Ar^2 - 2pr^4}}$ $= \dfrac{\sqrt{A}}{3} \dfrac{2r(A-4pr^2)}{2r\sqrt{A-2pr^2}} = \dfrac{\sqrt{A}}{3} \dfrac{(A-4pr^2)}{\sqrt{A-2pr^2}}$ $\dfrac{dV}{dr} = 0 \Rightarrow r^2 = \dfrac{A}{4p} \Rightarrow r = \dfrac{1}{2}\sqrt{\dfrac{A}{p}}$		4
1b (iii)	$A = 4pr^2$ and $A = pr(s+r) \Rightarrow 4pr^2 = pr(s+r) \Rightarrow s = 3r$ $s = \dfrac{3}{2}\sqrt{\dfrac{A}{p}}$ $h = \sqrt{s^2 - r^2} = \sqrt{9r^2 - r^2} = 2\sqrt{2}r = \sqrt{\dfrac{2A}{p}}$		3 **b =10**

QUESTION 2			
2a (i)	Both are rectangular hyperbolas		1
2a (ii)			3
2a (iii)	(i) H$_1$: $y + x\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x}$ H$_2$: $2x - 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{x}{y}$ Hence the result as $-\frac{y}{x}\frac{x}{y} = -1$		3 **a=7**
2b (i)	$x = ct,\ y = \frac{c}{t} \Rightarrow xy = ct\frac{c}{t} = c^2$		1
2b (ii)	Since $\frac{dy}{dx} = -\frac{y}{x}$ the gradient of the tangent is $-\frac{\frac{c}{t}}{ct} = \frac{-1}{t^2}$ Hence the gradient of the normal is t^2 Equation of normal $\frac{y - \frac{c}{t}}{x - ct} = t^2$ $y - \frac{c}{t} = t^2 x - ct^3$ $y = t^2 x - ct^3 + \frac{c}{t}$		4

2b (iii)	Gradient of TP: $$m=\frac{\frac{c}{p}-\frac{c}{t}}{cp-ct}\times\frac{pt}{pt}=\frac{ct-cp}{pt(cp-ct)}=\frac{-1}{pt}$$ Gradient of normal at T: $m=t^2$ So $$t^2=\frac{-1}{pt}\Rightarrow p=\frac{-1}{t^3}$$		3
2b (iv)	For midpoint $$x=\frac{1}{2}(ct+cp)=\frac{c}{2}\left(t-\frac{1}{t^3}\right)$$ $$y=\frac{1}{2}\left(\frac{c}{t}+\frac{c}{p}\right)=\frac{c}{2}\left(\frac{1}{t}-t^3\right)$$ Elim t $$\frac{2x}{c}=\frac{t^4-1}{t^3}$$ $$\frac{2y}{c}=\frac{1-t^4}{t}$$ Dividing $$\frac{y}{x}=-t^2$$ Substituting for t $$\frac{2x}{c}=\frac{\frac{y^2}{x^2}-1}{\left(\frac{-y}{x}\right)^{3/2}}$$ $$\frac{2x}{c}\left(\frac{-y}{x}\right)^{3/2}=\frac{y^2-x^2}{x^2}$$ $$\frac{4x^2}{c^2}\times\frac{-y^3}{x^3}=\frac{(y^2-x^2)^2}{x^4}$$ $$\frac{-4y^3}{c^2x}=\frac{(y^2-x^2)^2}{x^4}$$ $$4x^3y^3+c^2(y^2-x^2)=0$$		5 **b =13**

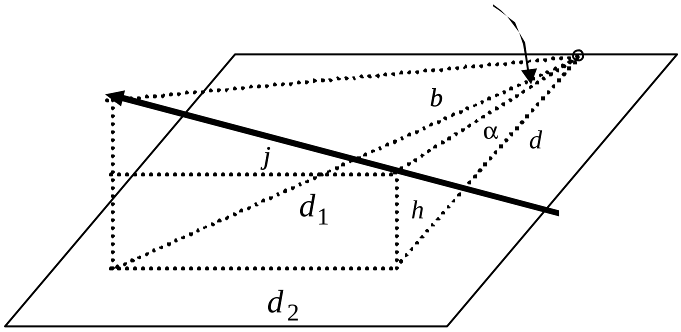
QUESTION 3			
3a (i)	$\frac{1}{x}+\frac{k}{A-x}=\frac{A-x+kx}{x(A-x)} \Rightarrow A=1$		2
3a (ii)	$y = Ae^{kbx}-\frac{a}{b}$ $\frac{dy}{dx} = Akbe^{kbx}$ $k(a+by)=k\left(a+b\left(Ae^{kbx}-\frac{a}{b}\right)\right)$ $= ka+kbAe^{kbx}-ka = kbAe^{kbx}$		3 **a=5**
3b (i)	Model B: $\frac{dn}{dt}=kn(P-n)$		1
3b (ii) **1.**	Model A: Setting $a = P$ and $b=-1$ in **a.**(ii) gives $n=-Ae^{-kt}+P$ $n=1$ at $t=0$ gives So $n = P-(P-1)e^{-kt}$ ie $(P-1)e^{-kt}=P-n$ ie $e^{kt}=\frac{P-1}{P-n}$		3
3b (ii) **2.**	Model B: $\frac{dn}{dt}=kn(P-n)$ $\int\frac{dn}{n(P-n)}=k\int dt$ $\int\frac{Pdn}{n(P-n)}=kP\int dt$ $\int\frac{dn}{n}+\int\frac{dn}{P-n}=kPt+c$ $\ln n-\ln(P-n)=kPt+c$ $\ln\frac{n}{P-n}=kPt+c$ $\frac{n}{P-n}=e^{kPt+c} = Ae^{kPt}$ $n=1$ at $t=0 \Rightarrow \frac{1}{P-1}=A$ $e^{kPt}=\frac{n(P-1)}{P-n}$		4

3b (iii)	Model B $e^{100000k} = \dfrac{20\times 19999}{19880}$ $:\Rightarrow k = \dfrac{1}{100000}\ln\dfrac{20\times 19999}{19880}$ $= 0.00003$ $e^{0.6t} = \dfrac{19999n}{20000-n}$		2
3b (iv)	Model B: $e^{0.6t} = \dfrac{19999\times 1980}{200} \Rightarrow t = \dfrac{1}{0.6}\ln\dfrac{19999\times 1980}{200} = 24.2$ ie 25 days		2
3b (v)	Model A: $e^{0.0002t} = \dfrac{19999}{20000-n}$ As $t \to \infty$ the LHS and hence the RHS $\to \infty$ so $20000 - n \to 0$ or $n \to 20000$ Model B: $e^{0.6003t} = \dfrac{19999n}{20000-n}$ As for model A $n \to 20000$		2
3b (vi)	Model B is likely to be the best model as the time taken for 99% to hear the rumour seems more realistic.		1 **b =15**

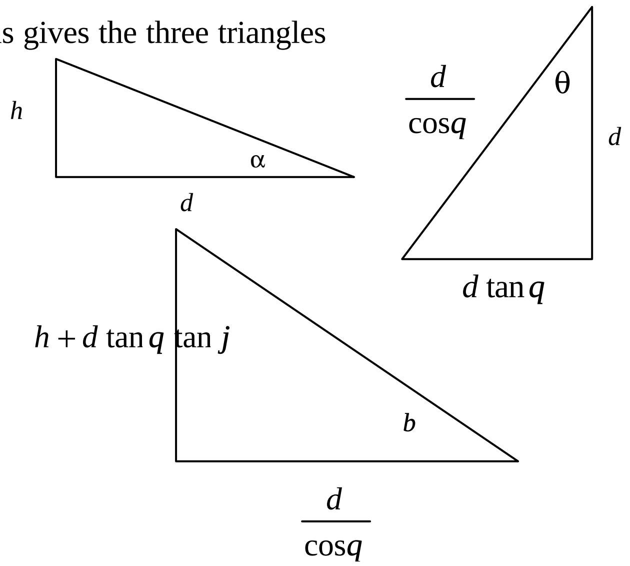
QUESTION 4			
4a (i)	$\sin 45 = \frac{1}{\sqrt{2}}, \quad \cos 45 = \frac{1}{\sqrt{2}}$		1
4a (ii)	LHS : $\frac{2\sin q \cos q}{1+2\cos^2 q - 1} = \frac{2\sin q \cos q}{2\cos^2 q} = \tan q$		2
4a (iii)	$\tan 22.5 = \frac{\sin 45}{1+\cos 45} = \frac{1/\sqrt{2}}{1+/\sqrt{2}} = \frac{1}{1+\sqrt{2}}$		3 **a =6**
4b	This gives the three right angle triangles shown  $h = d\tan 30$ $d_1 = \frac{d}{\cos 45}$ $h = d_1 \tan b$ so $h = \frac{d \tan b}{\cos 45}$ Then $d\tan 30 = \frac{d\tan b}{\cos 45}$ ie $\tan b = \cos 45 \tan 30$ or $b = 22.2^\circ$		7 **b=7**

4c

The angle is q



This gives the three triangles



So

$$h + d \tan q \tan j = \frac{d}{\cos q} \tan b$$

$$d \tan a + d \tan q \tan j = \frac{d}{\cos q} \tan b$$

$$\tan j = \frac{\tan b - \cos q \tan a}{\cos q \tan q}$$

Thus the angle of climb can be found by taking the inverse tan.

7

c =7

QUESTION 5			
5a (i)	$x=\dfrac{1\pm\sqrt{-3}}{2}=\dfrac{1\pm i\sqrt{3}}{2}=\dfrac{1}{2}\pm\dfrac{i\sqrt{3}}{2}=\cos\dfrac{p}{3}\pm i\sin\dfrac{p}{3}$		2
5a (ii)	$\dfrac{(cis\,2q)^5}{cis\,4q}=\dfrac{cis\,10q}{cis\,4q}=cis\,6q$		1
5a (iii)	$cis(-q)=\cos(-q)+i\sin(-q)=\cos q-i\sin q$ $(\cos q-i\sin q)^n=(\cos(-q)+i\sin(-q))^n$ $=\cos(-nq)+i\sin(-nq)=\cos nq-i\sin nq$		2 **a=5**
5b	$\dfrac{-1-\sqrt{5}}{2}\times\dfrac{-1+\sqrt{5}}{-1+\sqrt{5}}=\dfrac{1-5}{2\left(-1+\sqrt{5}\right)}=\dfrac{-2}{-1+\sqrt{5}}=\dfrac{-1}{g}$		2 **b=2**
5c (i)	8, 13		1
5c (ii)	$ax^{n+2}=ax^{n+1}+ax^n$ $x^2=x+1$ $x^2-x-1=0$ $x=\dfrac{1\pm\sqrt{5}}{2}=-g,\dfrac{1}{g}$		3
5c (iii)	$\left\lvert\dfrac{u_n}{u_{n+1}}\right\rvert=\left\lvert\dfrac{A(-1)^n g^n+\dfrac{B}{g^n}}{A(-1)^{n+1}g^{n+1}+\dfrac{B}{g^{n+1}}}\times\dfrac{g^{n+1}}{g^{n+1}}\right\rvert=\left\lvert\dfrac{A(-1)^n g^{2n+1}+Bg}{A(-1)^{n+1}g^{2n+2}+B}\right\rvert$ As $g<1$ all powers of $g\to 0$ as $n\to\infty$ $\left\lvert\dfrac{u_n}{u_{n+1}}\right\rvert\to\left\lvert\dfrac{Bg}{B}\right\rvert=g$		3 **c=7**
5d (i)	0, 1, 1, 0, -1		1
5d (ii)	$u_n=(C-iD)a^n+(C+iD)b^n=C(a^n+b^n)-iD(a^n+b^n)$ $u_n=C(a^n+b^n)-iD(a^n+b^n)$		1

5d (iii)	As $a = cis\frac{p}{3}$ $\quad b = cis\frac{-p}{3}$		
	$a^n = cis\frac{np}{3}$ $\quad b^n = cis\frac{-np}{3}$		
	$a^n + b^n = 2\cos\frac{np}{3}$ $\quad a^n - b^n = 2i\sin\frac{np}{3}$		
	$u_n = 2C\cos\frac{np}{3} + 2D\sin\frac{np}{3}$		4
	$u_{1=0:}$ $\quad 0 = 2C\cos\frac{p}{3} + 2D\sin\frac{p}{3} = 2C\frac{1}{2} + 2D\frac{\sqrt{3}}{2}$		
	$0 = C + \sqrt{3}D$		
	$u_{2=1:}$ $\quad 1 = 2C\cos\frac{2p}{3} + 2D\sin\frac{2p}{3} = 2C\frac{-1}{2} + 2D\frac{\sqrt{3}}{2}$		
	$1 = -C + \sqrt{3}D$		
	Adding: $1 = 2\sqrt{3}D$		
	So $D = \frac{1}{2\sqrt{3}}$, $\quad C = \frac{-1}{2}$		
	Hence $u_n = -\cos\frac{np}{3} + \frac{1}{\sqrt{3}}\sin\frac{np}{3}$		**d=6**

QUESTION 6			
6a (i)	$\frac{d \sin x\sqrt{1-9x^2}}{dx} = \cos x\sqrt{1-9x^2} + \sin x \frac{-9x}{\sqrt{1-9x^2}}$ $= \cos x\sqrt{1-9x^2} - \frac{9x\sin x}{\sqrt{1-9x^2}}$		3 **a=3**
6b	$\frac{d(uv)w}{dx} = w\frac{duv}{dx} + uv\frac{dw}{dx} = w\left(u\frac{dv}{dx} + v\frac{du}{dx}\right) + uv\frac{dw}{dx}$ $= uv\frac{dw}{dx} + vw\frac{du}{dx} + wu\frac{dv}{dx}$		2 **b=2**
6c (i)	$\frac{d\Delta}{dx} = \frac{-k^2 \sin x\cos x}{\sqrt{1-k^2\sin^2 x}} = \frac{-k^2\sin x\cos x}{\Delta}$		2
6c (ii)	$\frac{d \sin x\cos x\Delta}{dx}$ $= \cos x\cos x\Delta + \sin x.-\sin x\Delta + \sin x\cos x\frac{-k^2\sin x\cos x}{\Delta}$ $= \cos^2 x\Delta - \sin^2 x\Delta - k^2\frac{\sin^2 x\cos^2 x}{\Delta}$ $= \frac{(\cos^2 x\Delta^2 - \sin^2 x\Delta^2 - k^2\sin^2 x\cos^2 x}{\Delta}$		3
6c (iii)	$v = \sin^2 x \Rightarrow \quad \cos^2 x = 1-v$ and $\Delta^2 = 1-kv$ $\frac{d\sin x\cos x\Delta}{dx} = \frac{(\cos^2 x\Delta^2 - \sin^2 x\Delta^2 - k^2\sin^2 x\cos^2 x}{\Delta}$ $= \frac{1}{\Delta}\left[(1-v)(1-k^2v) - v(1-k^2v) - k^2v(1-v)\right]$ $= \frac{1}{\Delta}\left[1-k^2v^2 - v + k^2v^2 - v + k^2v^2 - k^2v + k^2v^2\right]$ $= \frac{1}{\Delta}\left[1-2(k^2+1)v + 3k^2v^2\right]$		3

6c (iv)	$\int \frac{dx}{\Delta} = F$ $\frac{v}{\Delta} = \frac{\sin^2 x}{\Delta} = \frac{1}{k^2}\left(-\frac{1-k^2\sin^2 x}{\Delta} + \frac{1}{\Delta}\right) = \frac{1}{k^2}\left(\frac{1}{\Delta} - \Delta\right)$ $\int \frac{vdx}{\Delta} = \frac{1}{k^2}\int\left(\frac{1}{\Delta}-\Delta\right)dx = \frac{1}{k^2}(F-E)$ $\int \frac{v^2dx}{\Delta} = \int v\frac{vdx}{\Delta} = \frac{1}{k^2}\int v\left(\frac{1}{\Delta}-\Delta\right)dx = \frac{1}{k^2}\left(\int\frac{vdx}{\Delta} - \int v\Delta\right.$ $= \frac{1}{k^2}\left(\int\frac{vdx}{\Delta} - \int(1-\cos^2 x)\Delta dx\right) = \frac{1}{k^2}\left(\int\frac{vdx}{\Delta} - \int\Delta dx +\right.$ $= \frac{1}{k^2}\left(\frac{1}{k^2}(F-E) - E + I\right)$ $= \frac{1}{k^4}F - \frac{k^2+1}{k^4}E + \frac{1}{k^2}I$ $\sin x\cos x\Delta = \int\frac{dx}{\Delta} - 2(k^2+1)\int\frac{vdx}{\Delta} + 3k^2\int\frac{v^2dx}{\Delta}$ $= F - 2(k^2+1)\frac{1}{k^2}(F-E) + 3k^2\left(\frac{1}{k^4}F - \frac{k^2+1}{k^4}E + \frac{1}{k^2}I\right)$ $= F - \frac{2(k^2+1)}{k^2}F + \frac{3}{k^2}F + \frac{2(k^2+1)}{k^2}E - \frac{3(k^2+1)}{k^2}E + 3I$ $= \frac{1-k^2}{k^2}F - \frac{k^2+1}{k^2}E + 3I$ $I = \frac{1}{3}\sin x\cos x\Delta + \frac{1+k^2}{k^2}E - \frac{1-k^2}{k^2}F$		7 **c=15**